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On Making Things the Best—Aeronautical Uses of Optimization

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Fear not to touch the best.
Sir Walter Raleigh, "The Lie"

Introduction

THERE is a lively branch of applied mathematics, which will here be called "optimization" and which attempts to choose the variables in a design process so as formally to achieve the best value of some "performance index" while not violating any of the associated conditions or "constraints." This discipline has its modern roots in work of Newton, the Bernoullis, and Euler. But such stories as Dido's founding of Carthage (see p. 11 of Wilde[1]) suggest that the Greeks knew a lot about it too. For an excellent historical survey, in the context of the closely related calculus of variations, see the little book by Bliss.[2]

The author confesses to a fascination with many aspects of optimization—an attachment perhaps not fully justified by the level of its current acceptance as a routine tool of engineering. Nevertheless, the published literature of the field is enormous. For example, a keyword search of just five files from Lockheed's DIALOG bibliographic indexing system, covering various periods between 1964 and 1980, yielded journal articles, reports, and dissertations in the following numbers: 4550 on optimal control, 2142 on aerodynamic optimization, and 1381 on structural optimization, for a total of 8073 aeronautically related titles. All this published research is complemented by surveys, texts, and reference books far too numerous to cite. Among these latter documents, Refs. 1-17 proved particularly interesting and helpful for preparing this paper.

As a goal appropriate to a diverse audience of aeronautical engineers, the one selected here is to assess the degree to which the "technology of the best" has contributed practically to the design and operation of vehicles which fly in the atmosphere. How this goal was approached will be described below, but Table 1 serves to emphasize the breadth of what was undertaken. There may be one or two omissions, but the table attempts to list the disciplinary areas of aeronautics wherein a significant number of applications have been published. The division into categories involving continuous or discrete mathematical formulation is convenient, albeit arbitrary and not necessarily inclusive. Thus the concept of "multistage" systems (see Bryson and Ho,[3] Secs. 2.2, 2.6, 6.10, and others) is intermediate between the two; any approximation to a continuous problem by numerical integration, digital realization, or series superposition in effect shifts it to the multistage or the discrete category.

The entries of Table 1 do not even embrace the whole of aerospace. Such purely astronautical applications as optimal orbit transfer and spacecraft guidance are bypassed because of the paper's aeronautical context. It is furthermore worth mentioning that formal optimization seems to have had a comparatively greater role in fields such as economic modeling[18,19] and chemical engineering[1,8,20] than in aerospace. This is probably because the near linearity of the input-output matrix and of the approximate relations describing refinery and chemical-plant operation facilitate the use of "linear programming,"[21] which was a very early triumph in the search for the best. Various branches of mechanical engineering (e.g., Ref. 22) and even recent automotive design[23-26] can be added to this partial enumeration of candidates for the applied optimizer.

Wilbur, Orville, and the Optimum

The Wright Brothers were well versed in the engineering mathematics of their time, but optimization as understood today would have held little attraction for them. One can imagine that they would soon have realized the dangers of powerful but obscure and unverifiable analysis applied to the shaky, imprecise data available to them regarding structural strength or the airloads on a wire-braced biplane. They might have anticipated a need for the sort of voluminous computation that is taken for granted today. They would have recognized the hopelessness of multivariate optimization without it. Nevertheless, the history of technology records few seekers of practical perfection who match these dedicated men.

Especially in the early days when the "Flyers" were evolving, the Wright papers (Ref. 27; see also Baker[28]) teem with instances where the finest available technology is pursued. All four pillars of aeronautical design are represented: control, structures, propulsion, and aerodynamics. Many writers (e.g., Combs[29]) have quite correctly highlighted the inspired attention to flying qualities, including effective control about all three axes of vehicle rotation. Yet the present author is equally impressed by the investigations on

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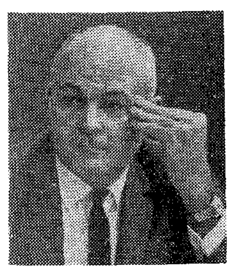

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Table 1 Aeronautical fields where formal optimization has been applied

Continuous representation (function space)	Discrete representation (vector space)
Flight trajectory optimization (single vehicle)	Wing shapes for maximum L/D, etc.
Combat encounter (two vehicles)	Optimal sizing of launch-vehicle stages
Guidance, navigation, control system optimization	Complicated structure of minimum mass, static design conditions
Airfoils and bodies of minimum drag, etc.	Complicated structure of minimum mass, dynamic, and aeroelastic conditions
Simple structure of minimum mass with static design conditions	Complete vehicle configurations for minimum mass, D.O.C., etc.
Simple structure of minimum mass, dynamic, and aeroelastic conditions	
"Distributed-parameter system" (e.g., two-dimensional structures)	

lifting surfaces and their adaptation to wing and propeller design. Everyone knows about the Wright's October 1901 wind tunnel. Not so familiar is that this instrument was used for accurately measuring the variations with angle of attack of lift L and drag-lift ratio D/L on nearly sixty wing models—in monoplane, biplane, and triplane combinations. These models are described on pp. 550-573 of the papers[27]; measured data appear in Tables I, II, and III of Ref. 27. Camber, aspect ratio, and other aerodynamic parameters were varied in a systematic fashion.

Of the model tests Wilbur says in his Dec. 15, 1901, letter to Octave Chanute, "A study of the series plates quickly discloses the general principles which govern lift and tangential* and thus renders the search for the *best* shapes much easier." Putting their philosophy into practice, the brothers selected Model No. 12 as the basis for the design of their 1903 "Flyer." With aspect ratio 6 and 5% camber, this model attained a maximum L/D of about 9.5 at 5-deg angle of attack. It indeed achieved the "highest dynamic efficiency of all the surfaces," as reported by Wilbur. From some 510 test points in Table II of Ref. 27, this L/D is the greatest. Could not the choice of this wing then be characterized as optimization by exhaustive search of parameter space?

On Plates 131-141 and pp. 594-640, Ref. 27 tells the extraordinary story of how the Wrights developed and applied what amounts to the first combined momentum and blade-element theory of propellers. This work resulted in a propulsion system of unparalleled mechanical efficiency. For instance, on the 1907-1909 series of machines, with the familiar large-diameter, counterrotating propellers, the ratio of thrust power to engine brake power has been estimated in cruise at between 71 and 76%. Koppen[30] has stated that these values exceed the efficiency of any other aircraft propellers flown prior to World War I. On their models of the period 1909-1911, the brothers even incorporated a modest sweepback near the tips. Thus they adapted a favorable aeroelastic phenomenon to minimize twisting under load.

Some Generalities: The Program of This Paper

Figures 1a and 1b present one viewpoint of how optimization might fit into the practice of aeronautical engineering. For purposes of this morphology it seemed convenient to make a separation between the optimum design of a new vehicle or portions thereof, on the one hand, and the best way to employ an existing vehicle or system on the other. Up to a point in the process, Fig. 1a merely summarizes how any engineering team would go about meeting a specification or developing operational procedures.

No self-respecting designer likes to admit that his product is not, at least in some pragmatic sense, the best. Proposals and tradeoff studies are full of charts which show the chosen configuration to fall at the peak of an efficiency curve or the valley of weight or cost plotted vs some key parameter. These curves are, in fact, the results of one-dimensional optimizations. The engineering decision makers' lot would be much easier if a sequence of such unidirectional searches usually generated a good approximation to a single multidimensional search over all the important independent variables. Unfortunately, this is often not the case, as illustrated by Schmit's[31] findings about fully-stressed design.

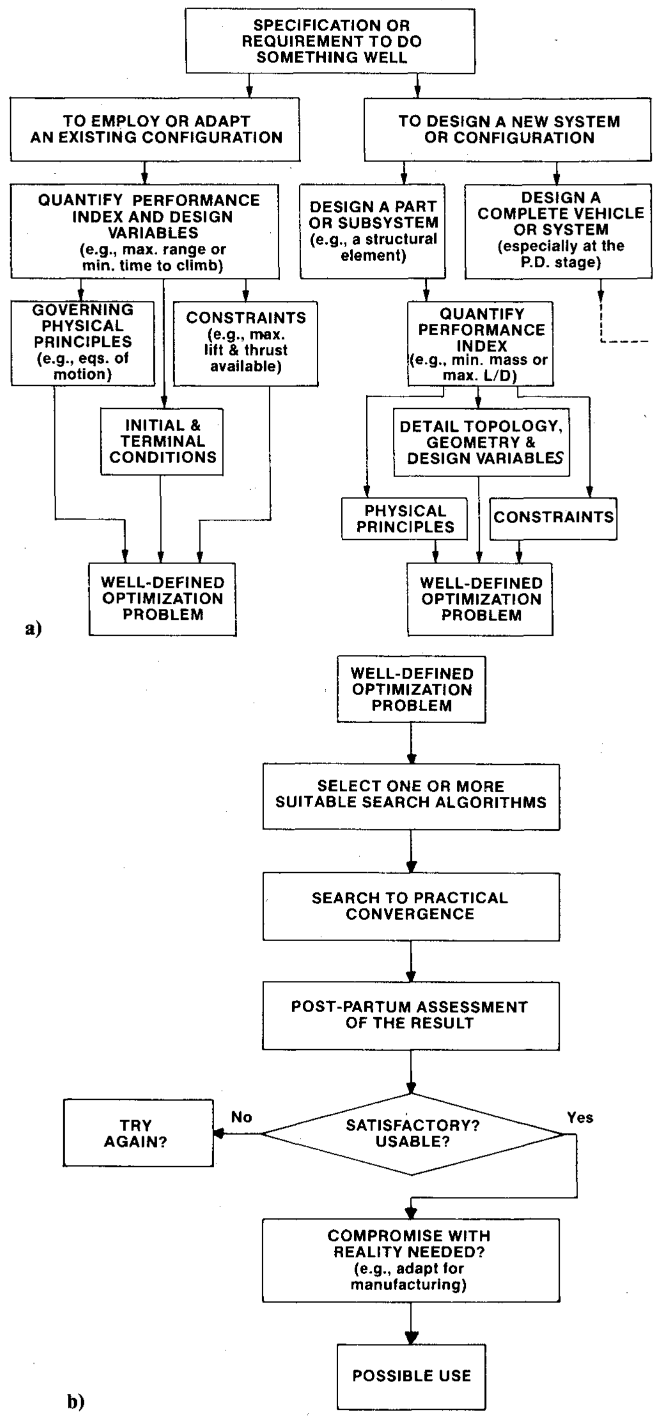

Fig. 1 Morphology of optimization in aeronautics.

*"Tangential" was a term used for the angle indicated on the Wright's drag-lift balance. When added to angle of attack, it gives the equilibrium glide angle or $\tan^{-1}(D/L)$.

At times formal optimization yields highly counterintuitive discoveries. Certain examples discussed below demonstrate that these surprises may occasionally have a certain utility.

If a commitment is made to the formal approach, however, then at some juncture the job must be reduced to a determinate mathematical statement. The author's investigations reveal that this may be the stage where engineering judgment and experience attain their greatest importance. It is definitely not just a matter of rolling everything up into a welter of equations and letting the computer grind out the answers. In the case of structures, for instance, the topology and general layout of members, boundaries of the volume to be occupied, the choice of material systems, the modes of failure, and the loading conditions likely to be most significant are all matters which are usually addressed as part of the formulation rather than the solution. The most appropriate performance index is not always obvious, especially in connection with vehicle operation or preliminary design of a new system.

Further mention will be made in what follows of the keen disappointment felt by many specialists because their theories have received so little practical application. This phenomenon is frequently attributed to a reluctance by developmental engineers to adopt unfamiliar and untried methods of analysis. Accepting this as a partial explanation, one can further speculate that the difficulty of efficiently formalizing a realistic design or operational question may be another source of discouragement. Today it requires much more knowledge and understanding on the user's part to exercise successfully a package of optimization software than, by comparison, to stress-analyze a complicated structure by means of a finite-element code. Furthermore, it often turns out that failure to enforce properly even one or two key constraints may lead to pretty outlandish results.

It is hard to find a better summary of the statement and solution of discrete-variable optimization problems than Schmit's 1977 AIAA Structures Design Lecture.[32] Among the set of quantities needed to define a system, he first designates as "preassigned parameters" those that are fixed at the outset. The "design variables" (called in different contexts by other names; e.g., "control variables") are those quantities that are changed during the modification procedure which seeks an optimum. These N real numbers are conveniently written as an $N \times 1$ vector D. Recognizing that only a single scalar† can be optimized at a time, one must devise a performance index M, which is a single-valued function of D. Because of the possibility of changing its sign or inverting, M can always be chosen so that the goal is

$$M(D) \rightarrow \text{minimum}$$

The search must be carried out in an N-dimensional space populated by Q barriers, which quantify the constraints. At least "in a rather general class of structural synthesis problems," Schmit observes that these can be formulated as Q functional inequalities,

$$g_q(D) \geq 0, \qquad q = 1, 2, \ldots, Q$$

Certain of the functions g_q may be as simple as linear relations that enforce minimum- and maximum-gauge constraints on the cross-sectional dimensions of members. In aeronautics, however, the set is almost sure to contain some complicated and perhaps implicit functions of D, demanding considerable machine computation for their evaluation. With structures again used for illustration, there may be constraints on strength and stability associated with failure modes under various conditions of loading; limits on deflection at specified stations; lower bounds on frequencies of free vibration; requirements about the avoidance of such aeroelastic instabilities as flutter; etc.

Lest this discussion seem too slanted toward the structures field, however, it is noted that several other examples in Table 1 can be cast into a similar format. Thus one might seek to apply linearized aerodynamic theory for synthesizing the paneled camber surface of a wing of given planform flying at given speed and altitude. The performance index might be L/D at a prescribed lift coefficient C_L. Inequality constraints might be placed on the percentage camber and the distribution of twist, on the pitching moment at zero lift because of trim considerations, and perhaps on the camber-line slopes along the leading and trailing edges.

In Fig. 1a a distinction is made between governing physical principles, like the laws of statics and dynamics, and constraints per se. Both of these are brought together in the development of the more complicated among the constraint functions g_q. It is only in such trivial instances as minimum-gauge that a constraint is just the arbitrary result of an engineering judgment.

Schmit[32] and all of the previously cited textbooks contain catalogues and comparisons of the "various approaches employed in the quest for improved and/or optimum designs." It will come as a surprise to some readers that the present paper devotes little attention to that crucially important subject. One acknowledges vast opportunities for both fundamental research and practical improvements. Here, however, the viewpoint is adopted that, once a problem is reduced to feasible magnitude and properly posed, accurate and efficient means will be at hand to compute at least a locally optimal solution. Enough surveys, competitive analyses of test cases, and the like are in the literature to justify this claim. Furthermore, it is known that a certain amount of optimization software is available to the engineers at *every* major airframe manufacturer in the free world. Skilled specialists are also on hand, if only they are called upon, to assist with applications far more numerous than those actually carried out today.

Figure 1b completes the summary of steps begun in Fig. 1a. It is believed that they are similar enough among the classes of optimization identified in Fig. 1a that only a single sequence serves to cover all three. If one assumes a well-formulated synthesis task and a satisfactory search, then engineering judgment and experience must again be invoked to evaluate and adapt the optimum vector D_{opt}. A mathematical issue that is sometimes slighted in practice concerns whether D_{opt} is a true minimum for a function $M(D)$ and whether that minimum is "global." There are well-established tests associated with the second variation of M (see Chap. 7 of Bryson and Ho[3] or Chap. 15 of Leitmann[4]). Being generally difficult to apply, however, these criteria are frequently ignored when the design looks reasonable and useful. It is left to the experts to argue the merits of such an oversight.

More significant for the working engineer are the practical assessment, the checking, and the review of his brainchild by superiors on the design team. If these "gates" are successfully passed, then one expects that D_{opt} will become part of the configuration or be incorporated into operational procedures in the same way as any improvement generated by other logical means. It must also survive the gauntlet of ground testing, reliability demonstration, flight verification, and the like without further special attention.

One additional remark is offered on the subject of rigor. Perfectionists among those specializing in the field deplore using the word "optimal" to describe the results of searches wherein certain mathematical formalities are disregarded and/or such convenient approximations as variable-linking are employed to speed convergence or to reduce the volume of computation. Indeed, this is more than just a question of semantics. The operative facts for engineering utility, however, are that designs produced in such "suboptimal"

†The concept of "multicriteria optimization," see Stadler,[33] would seem to contradict this statement, but in fact what it appears to do is construct a single performance index by some functional combination of quantities representing benefits and disbenefits.

ways are often superior to those produced by standard methods, notably in cases where rigorous optimization might be impractical. In structures, for instance, even putting together a sequence of fully-stressed designs for multiple loading conditions—a practice which has been discredited, and rightfully so, when there is a great deal of indeterminacy (see Petiau and Lecina[34])—may furnish a very satisfactory alternative to the trial-and-error product of the stress-man at his board or console.

As a preliminary to writing this paper, the author scoured available sources in the hunt for genuine (vs hypothetical) aeronautical instances of applied optimization. Memory and the literature were, of course, exhaustively scanned. Forty-six friends, colleagues, and specialists in the technology received two-page letters, with return forms requesting data on "successful practical applications...to aeronautical problems." Innumerable telephone calls solicited further examples or details regarding cases already in hand. Occasionally, but not to an extent that would affect the principal conclusions, roadblocks were encountered owing to military classification or company proprietary considerations.

In a word, the response to this survey proved overwhelming. The expressions of interest and encouragement were a welcome stimulus. More than three-quarters of those contacted replied in extenso. But the yield of examples which met the criterion of having been incorporated into a vehicle that actually operates in the Earth's atmosphere was painfully, perhaps shockingly small. The original intent was to require that every example summarized in the main body of the paper pass this test. Some compromise has been necessary, as will be seen. Nevertheless, more than half of all genuine cases discovered in the literature or the survey are included here. (This count excludes a few specialized applications like the use of linear quadratic synthesis for state estimation.)

Some quotations from typical respondents may help readers to understand the dilemma:

> From an engineer experienced in civil and aeronautical structures, "One of the reasons that I stopped work in optimization was my dismay... that there were so very few applications."
>
> From a Dean of Engineering who has known the field for over a quarter century, "I do not recollect any applications."
>
> From a foremost specialist on synthesis with aeroelastic constraints, "I am sorry, but I don't really have any...."
>
> From a recently retired senior design engineer, describing events at his aerospace company, "For 15 years I beat my head against a stone wall... The end was: formal optimization techniques were never used in aircraft design (even to this day!). The company was forced to use [them] in its subsequent ICBM and space programs."
>
> From an interview with a distinguished European aerodynamicist regarding wing design, "Numerical optimization techniques were being used to some extent in prefeasibility studies, but he was unable to produce any specific or referenceable material."

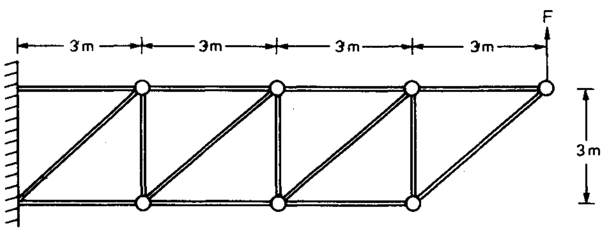

Fig. 2 Planar, pin-jointed truss used to illustrate optimal structural design. Each bar has solid circular section. Single load *F* applied at tip.

The obverse of this tarnished coin contains two bright spots. First, one can experience true excitement from studying several of the cases that have been unearthed. Unforeseen solutions reward the dedicated optimizer, as do occasional dramatic gains over the traditional approach. Second, there are countless "paper" investigations, for which citations are omitted here because of space limitations, but whose promise for improved efficiencies of many different kinds deserves far greater recognition by line engineers than it has received hitherto.

A Few of the Possibilities, as Explained with a Simple Truss Design

With the valuable help of Stanford doctoral candidate P. Hajela, the author has explored some variations on an elementary problem in structural synthesis. The aim is to illustrate, for the nonspecialist, typical aspects of optimization. This approach and the choice of an initially determinate truss were inspired by a similar study[35] made once by Pedersen for the AGARD Structures and Materials Panel.

The planar truss of Fig. 2 might be thought of as symbolizing half of an airplane wing, with the distributed lift replaced by an "equivalent" force F near the tip. The fourteen bars have solid circular cross sections and are connected to one another and to a rigid base by ideal pin joints. The material is aluminum alloy with density 2.8×10^3 kg/m^3 and limit stress 2.45×10^8 N/m^2 in both tension and compression. As in many structural applications, total mass is to be minimized through the choice of the member areas while insuring no failures under the given loading.

The analysis is simplified by assuming that the effects of nodal displacements on internal load distribution are negligible. Consider first a high tip force, $F = 2 \times 10^6$ kg or 19.6×10^6 N. The original structure is nonredundant, so the tension or compression in each member can be calculated by standard methods (see Chaps. 6 and 7, Housner and Hudson[36]). It is obvious that the lowest mass is achieved by working every bar at its limit or allowable stress. As shown in Fig. 3, the corresponding total is 18,200 kg.

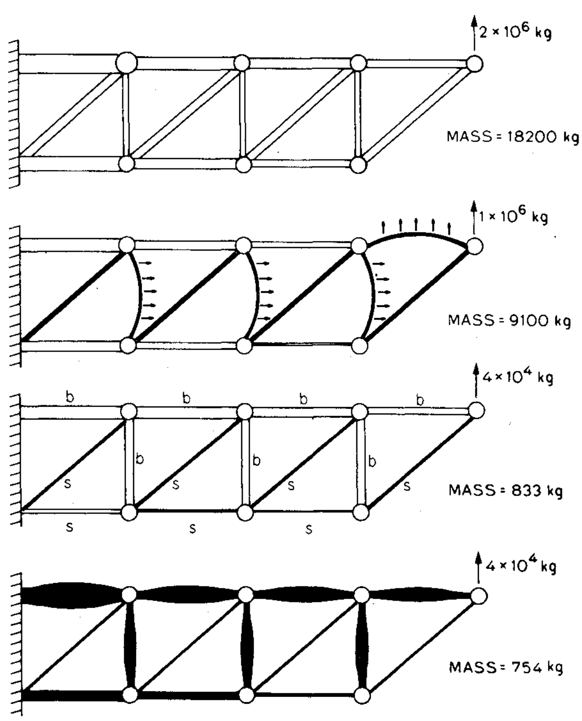

Fig. 3 Four minimum-mass trusses with different tip loads. (See text for details. Bar thicknesses give only a qualitative sense of cross-sectional dimensions.)

This is an example of fully stressed design. Such cases of uniform strain energy are often called "Michell structures," after the author who introduced the concept of the optimal frame to support a single set of loads (Ref. 37; see also Hemp[38]). One should remark, however, that Michell's investigation actually relates[38] to patterns of "tension and compression members continuously distributed through a region" or prescribed bounding volume. By thus encompassing the layout and topology of the structure as well as individual member areas, he goes well beyond simple trusses composed of a preassigned set of rectilinear bars.

Returning to the present example, one imagines F to be gradually reduced, with the optimal bar areas and the total mass decreasing in direct proportion. This process leads through a sequence of meaningful optima until F falls to about 10^6 kg, whereupon an event depicted in the second sketch of Fig. 3 takes place. Here the four members carrying the lowest *compressive* loads reach the Euler buckling limit for a pin-ended column. The design's validity begins to be endangered by the appearance of a linear, elastic instability.‡

It is incidentally remarked that the solid members' ability to withstand compressive stress depends on their areas A, whereas their buckling loads are proportional to A^2. Therefore reducing F moves each optimal compression member in the direction of instability. This is why more efficient thin-walled cylinders were not used in the example; such a truss would have its elements sized always by strength or always by buckling, independently of the magnitude of F.

Let F be fixed at 40×10^3 kg for the rest of the cases. As shown in the third sketch, *uniform* bars whose areas are determined either by buckling (marked b in Fig. 3) or by strength (all others) have a total mass of 833 kg. The next refinement is to replace those uniform b-bars by members of variable circular cross section but determined, by distributed-parameter optimization, to have minimum mass for given compressive load with combined buckling and strength constraints. The Appendix supplies the details of this design, which results in a partially tapered configuration having maximum diameter at midlength. The new optimal elements allow a further reduction in the truss's mass to 754 kg—a 9.5% improvement, but one to be paid for by the added cost of manufacturing the noncylindrical shapes.

Figure 4 presents four more attempts at refinement, two where additional members are inserted and two where the four lower nodes are allowed freedom to move vertically. Four more design variables obviously enter the calculation in the latter case. Since the trusses with extra bars are indeterminate structures, something more complicated than the foregoing approach is required. The search procedure actually employed was one wherein a nonlinear programming algorithm was associated with finite elements in a format suggested by Sobieski and Bhat.[39] The cumulative constraint formulation of Hajela and Sobieski[40] was also adopted. Piecewise linear approximations were applied to the performance index and the constraint values in order further to simplify computations.

The best of these four improvements bring the mass down near 700 kg. There is an interesting consequence of adding three redundant diagonal bars: when the optimum is reached, one of the original bars (dashed in Fig. 4) turns out to make an insignificant contribution to the structure and can therefore be omitted. In the lower two cases where nodes are given freedom to move, only rather small displacements occur from the original positions. This is because the reference layout is already quite efficient. As the nodes proceed downward, the capacity to carry bending moment increases as a consequence of greater depth. But the inevitable tradeoff soon limits their excursions because the members in compression lose effectiveness on account of their increased length.

‡It is well known that to permit simultaneous instability of two or more members yields an unsatisfactory design, very sensitive to imperfections. This fact does not, however, destroy the usefulness of the truss as an illustrative example.

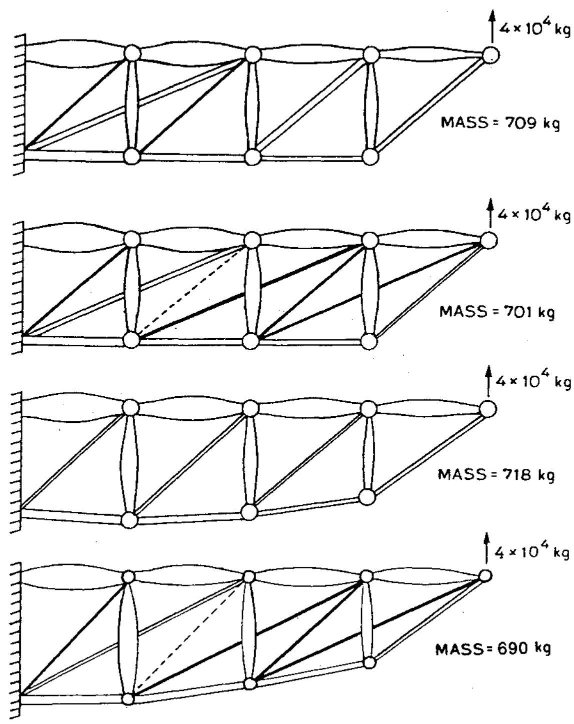

Fig. 4 Four minimum-mass trusses, with same tip load but different configurations. (See text for details.)

Actual and Potential Aeronautical Applications

Prize that which is best in the universe; and this is that which useth everything and ordereth everything.

Marcus Aurelius, "Meditations," v. 21.

Aerodynamics and Propulsion

The book edited by Miele[6] remains both a useful text and survey, as of 1965, on aerodynamic optimization. It is a pity that elegant and often practical solutions of the sort which populate every chapter have contributed so little to applied aeronautics. For instance in Chap. 12 of Ref. 6, Hayes summarizes what was probably the first "modern" optimal design: Newton's use of his famous formula, giving the pressure p on a surface inclined to an airstream, for determining the body of revolution with minimum pressure drag, fixed length l, and fixed base diameter. His approach helped to establish the foundations of variational calculus.

Hayes points out that the Newton formula is, in effect, a measure of momentum exchange with a diffusely reflecting cold body and is a good empirical approximation for the windward portions of convex shapes in hypersonic flight. It reads as follows in terms of atmospheric pressure and density p_∞ and ρ_∞, airspeed V, and angle θ between the flight direction and the local normal to the body surface:

$$p - p_\infty = \rho_\infty V^2 \cos^2\theta \qquad \text{(N1)}$$

Eggers (Chap. 6 of Ref. 6) plots in his Fig. 4 the dimensionless contours of the optimum bodies for five values of fineness ratio. His review also contains key references and "Newtonian" shapes appropriate to other constraints, like prescribed diameter and volume.§ One curious finding is that

§Eggers et al.[41] published a definitive paper, whose measured data verify clearly the superiority of the optimum shapes over cones, etc. Their mathematical approach is through classical calculus of variations.

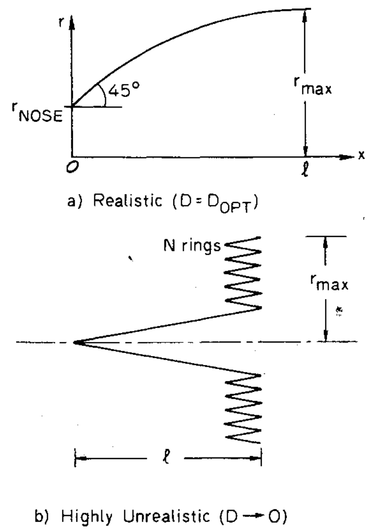

Fig. 5 Two minimum-drag bodies of revolution, designed for fixed ratio of length to base radius by Newtonian impact theory. In part b, unrealistic strong variations in $r(x)$ are permitted.

the original family all have flat forward faces, which are met at a 45-deg angle by their curved portions, as in Fig. 5a. This is in contrast with other optimal hypersonic figures and with linearized-theory results for lower Mach numbers, most of which are pointed at $x=0$.

In addition to their historical significance, it seems clear that these solutions have played at least a qualitative part in the design of hypervelocity rifle bullets. Their derivation also nicely illustrates the requirement for engineering judgment to avoid mathematically correct but meaningless distractions. Thus, if one omits the flat face, the drag D corresponding to Eq. (N1) for a body of radius $r(x)$ can be written in the alternate forms

$$\frac{D}{2\pi\rho_\infty V^2} = \int_0^l r\left|\frac{dr}{dx}\right|^3 dx \Big/ \left[1+\left(\frac{dr}{dx}\right)^2\right]$$

$$= \int_0^{r_{max}} r\left|\frac{dr}{dx}\right|^2 dr \Big/ \left[1+\left(\frac{dr}{dx}\right)^2\right] \tag{N2}$$

Suppose that one does not require dr/dx to be monotonic or non-negative and further admits of "strong variations" when applying the calculus. One can then devise a jagged body (Fig. 5b) with a spiked nose and N V-shaped, concentric rings resembling a Fresnel lens. It can be adjusted, for large N, so that on each ring

$$\left|\frac{dr}{dx}\right| \sim N^{-1} \tag{N3}$$

Then in the limit as $N \to \infty$, one will find that Eq. (N2) yields zero drag!

Easy to overlook but nevertheless an example of formal optimization is the next aerodynamic discovery in historical order: the untwisted wing of elliptical spanwise chord variation, which is predicted by lifting-line theory (Prandtl, Sec. IID, Ref. 42) to develop minimum induced drag for a given lift at all angles of attack. Prandtl's planform had obvious influences on the design of early monoplane wings, the Boeing P-26, Supermarine Spitfire, and Republic P-47 Thunderbolt being among the easiest to identify. The second of these fighters, whose maiden flight occurred in March 1936, was described at the time[43] as "the fastest military airplane in the world." It later became one of the technological heroes of the Battle of Britain. The Spitfire evolved from its manufacturer's experience with Schneider Trophy competitors. It boasted many advanced features, including stressed-skin aluminum construction that achieved a remarkably low structural-weight fraction.

The elliptic planform is less producible than a developable-surfaced wing. It is therefore convenient that the unswept, trapezoidal shape with approximately 2:1 taper ratio can get nearly the same aerodynamic performance. Many subsonic aircraft of the decades following 1930 took advantage of this fact.

In low-speed propeller theory, there is an optimally efficient analog to the elliptic wing. This is Goldstein's solution,[44] whose helical vortex wake is predicted to move backward through the fluid as a rigid surface just as the elliptic wake is convected vertically in planar fashion by its own downwash. Despite an effort by Theodorsen[45] to extend the Goldstein design from lightly to heavily loaded propellers and to encourage its use, the author has found little evidence of its influence on configurations in actual operation.

Subsonic, transonic, and supersonic potential-flow aerodynamic theory has supplied the analytic foundations for optimization studies on fuselages, wings, etc., in a variety of circumstances. One of the earliest such results is the "Sears-Haack body," which is the doubly-pointed axisymmetric shape of minimum supersonic wave drag for given length and volume (see Sec. 4.1, Chap. 7 of Ref. 6). Although details have not been published, some variant of this body influenced[46] the evolution of Convair's series of delta-wing fighters. It is documented[47] that the "area" or "equivalence rule" (e.g., Whitcomb[48]) was able to cut transonic drag rise almost in half when used to improve the sectional area distribution $S(x)$ of the prototype F-102. By narrowing the fuselage along the wing root and adding a bulge near the trailing edge, Convair produced the F-102A. It was when F-102A was modified into the even-higher-performance F-106, however, that aerodynamicists[46] employed results like Sears-Haack as a qualitative standard of reference for further revising $S(x)$ toward the optimum. Both transonic and supersonic area rules (see, e.g., Lomax and Heaslet[49]) at several Mach numbers up to two provided effective $S(x)$ distributions for comparison purposes. F-106 remains the backbone of U.S. manned continental air defenses today.

Since about 1968, codes have been available which, given the motivation, could serve to synthesize lifting surfaces and wing-body combinations under various choices of performance index and practical design constraints. Typical of the first generation is that described by Hague et al.[50] Although it is asserted that these methods were involved in such diverse applications as to improve the stall characteristics of Cessna's T-210L Turbo Centurion and to shape a proposed wing-root fairing for the stretched C-141B transport, no documented example has been found of a fully optimized wing installed on an operating vehicle.

Among comprehensive studies which have recently been reported relative to lifting surfaces, two are chosen as representing the possibilities of the formal approach. Haney and Johnson[51] developed and wind-tunnel tested two candidate wings to improve transonic drag of the A-7 attack aircraft. On a preselected planform at given Mach number and incidence, their procedure adapts an optimization due to Hicks and Henne.[52] It works by revising sequentially the airfoil sections at several spanwise stations so as to minimize the error-squared between prescribed, favorable chordwise distributions of upper- and lower-surface pressures and actual pressures predicted by a three-dimensional transonic code.[53] Design variables are twist angle, trailing edge camber, and the coefficients of twenty-two shape functions at each of five locations on the semispan. Other details are furnished in Ref. 51.

Figure 6 shows the projected shapes and other information on the original A-7 wing, the first alternative which has a nearly identical planform, and the second whose aspect ratio is increased from 4 to 5. Figure 7, picked from the many types of data available in Ref. 51, summarizes for two lift coefficients the drag reductions and "drag-break" Mach numbers achieved by the two redesigns. No decision has been made to incorporate such modifications into A-7 production or to replace wings of existing aircraft.

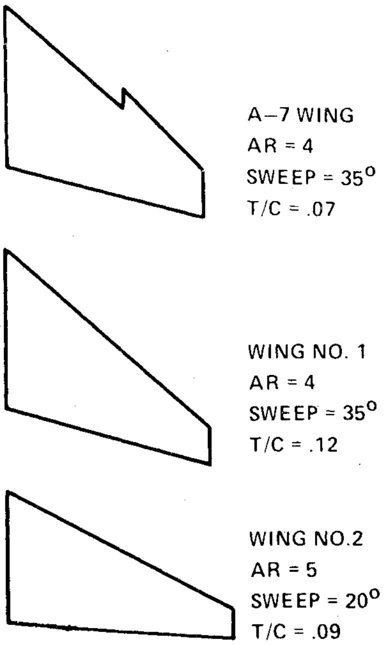

Fig. 6 **Plan views and other details for original A-7 wing and two reduced-drag substitutes designed by Haney and Johnson (Ref. 51).**

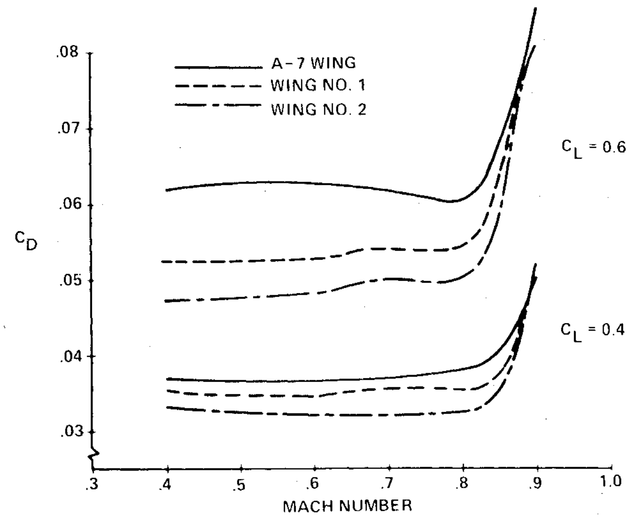

Fig. 7 **Dependence of drag coefficient on Mach number, measured at two values of lift coefficient for the wings of Fig. 6.**

References 51 and 52 and similar numerical schemes seem likely to become a significant tool for preliminary design of unconventional vehicles which require maximum performance in the vicinity of Mach 1. Obvious candidates are the forward-swept (FSW) fighters under development by several manufacturers for USAF and DARPA. Wilkinson[54] mentions a "drag minimization procedure which has been used extensively by our Aerodynamics Group on the... program and, also, on the CDAF and VTX programs." Outlined in a Grumman memo,[55] it adapts Lamar's vortex lattice theory[56] to estimate profile as well as induced drag. For a close-coupled pair of surfaces like the wing and stabilizer of a FSW, it minimizes total drag for specified lift, trimmed c.g. location, etc. One result quoted in Ref. 55 is a 40-count drag reduction at Mach 0.9 and $C_L \cong 0.9$ due to reshaping a typical FSW research model.

Aerospace propulsion systems, because of their complexity and the numerous variables available for selection by their designers, would seem to offer another rich field for applied optimization. Opportunities arise both during the initial aerodynamic and structural configuration of a new engine and in connection with improving the control and efficiency of an existing machine. Among the many interesting theoretical studies that have been published, one can cite a comparative evaluation[57] of nine different procedures for predicting the best vane and bleed settings in a multistage axial compressor. The authors incidentally found distinct advantages for Vanderplaats' CONMIN[58] method of feasible directions—the same code that was used in Ref. 51.

Recently Rao and Gupta[59,60] examined the minimization of aerodynamic losses in one stage of an axial gas turbine. Also by means of CONMIN, Hearsey[61] demonstrated how savings ranging between 5 and 15% might be achieved in the energy consumption of the NASA Ames Research Center 11-ft Transonic Wind Tunnel. Some of his recommendations and a program ACL300 which he prepared were, as of Spring 1981, being applied for rebuilding the stators and otherwise modifying the tunnel compressor.[62]

As for large liquid rocket motors, a report by Hague et al.[63] appears to represent the first attempt at using optimal search to alleviate combustion instability while attaining high performance. It is said to have been employed during the assessment of competing designs for the Space Shuttle Main Engine.

In 1977 De Hoff et al.[64] reported on the first phase of a comprehensive investigation aimed at designing and testing advanced digital controls for the F100 turbofan engine, which powers the F-15 and F-16 fighters. As can be inferred from Table T-1 of Hill and Henderson[65] there are difficulties with transient and cyclical behavior of the current F100 that make it an attractive candidate for sophisticated control enhancement. Based on the optimal concept of the linear quadratic regulator (LQR; see Bryson and Ho,[3] Sec. 5.4), the Ref. 64 system aims for both efficient steady-state operation and smooth transients within constraints set by stall, surge, permissible temperatures, and the like. In addition to demonstrations on simulators, it was successfully applied to a full-scale F100 in an altitude test cell at NASA Lewis Research Center. It also serves as the foundation for ongoing work by the engine manufacturer and various airframe contractors, which one hopes may ultimately lead to significant performance and life improvements for well over one thousand F100's now in the inventory.

Flight Trajectories

Aircraft trajectory optimization formed one of the earliest foci of modern research, and it remains a productive contemporary field. Prior to World War II, work centered on minimum consumption of fuel or minimum dollar expenditure during the cruise portions of commercial flights. The post-war history of small-scale attempts to do this for piston-engined aircraft has been summarized by McLean.[66] Optimum flight planning ceased to be a matter of hand calculation, however, when jet propulsion was introduced (1958 in the U.S.). The new airplane-engine combinations had the property that altitude for maximum true airspeed no longer coincided with that for minimum fuel flow—not to mention difficulties with meteorological conditions near the tropopause.

Then Director of Flight Performance for American Airlines, McLean[67] was responsible for implementing the first computerized optimum flight planning. The original American system built up about 200 domestic routes from 220 point-to-point segments. Forecast temperature and wind data were stored so as to permit the synoptic calculation of roughly 100 plans for each city pair, involving ten or more altitudes and several ground tracks. The three best were identified and offered as a choice to the flight crew. Of numerous available performance indices, minimum direct cost subject to meeting schedule was the one most commonly specified. Taking advantage of favorable winds was the largest single cause of improvements.

McLean[66] estimates that by 1965 this innovation was already saving his employer some three million dollars yearly in fuel alone. Such economies have, of course, grown many fold as energy prices rose and other airlines embraced optimization. R. Dixon Speas Associates pioneered automated optimal flight planning for airlines operating over the North Atlantic. Their use of dynamic programming (see Ref. 68; also Ref. 3, pp. 131-147) increased the efficiency of the required computations over what had previously amounted to exhaustive search.

Until recently formal methods were concentrated on the cruise portion of the airliner's trajectory, leaving the climbout and descent phases as relatively short segments where Air Traffic Control tends to supervise the flight anyway. For cruise the classical range equation, modified for a jet operating at fixed L/D ratio in a constant-temperature region of the stratosphere, gives the distance traveled through the air as

$$R = V(L/D) I_{sp} \ln(W_0/W_1) \qquad \text{(R1)}$$

Here V is the true airspeed, I_{sp} is the specific impulse in seconds, and W_0, W_1 are vehicle weights at the beginning and end of the segment, respectively.

Conventional wisdom (see p. 259, Ref. 69) accepts the implications of Eq. (R1) that a rectilinear path flown near the peak value of $V(L/D)I_{sp}$ will always maximize the range for a given amount of fuel consumed (no wind). This hypothesis was recently shattered, however, by an amusing and highly counterintuitive discovery due to Speyer (Ref. 70 and others). By second-order variational analysis he identifies, under reasonable assumptions on vehicle drag and I_{sp}, circumstances where "steady-state cruise for a long time span is *nonoptimal.*" In Ref. 70 it is suggested that the least-fuel path may be periodic. Because it is expected to constitute "a large variation in state away from the cruise arc," no truly extremal solutions are presented.

Breakwell and Shoaee[71] went a step farther in 1980. Assuming constant I_{sp}, and typical dependence of drag on speed and altitude,¶ they calculated tracks involving sinusoidal variations of speed, flight-path angle γ, and other parameters which yield up to 3% advantage in fuel per unit distance over steady "optimal" cruise. Figure 8 is adapted from Ref. 71 to show the characteristics of a sample case. The oscillation has a period of 4.7 min, slightly over twice the airplane's *phugoid* period at a best speed of 296 m/s near the tropopause ($h_c = 10$ km). The required changes in thrust probably exceed what the engines are capable of. Even if the passengers did not complain of accelerations varying ±15% from 1 *g*, it seems likely that traffic controllers and those responsible for gas turbine maintenance would object to such deviations from the operational norm!

There are at least two respects in which flight planners might improve their art beyond the level represented by Refs. 67 and 68: by introducing more efficient search algorithms and, as mentioned above, by encompassing climbout and descent in the optimization. The latter innovation is attractive for short- and medium-haul flying, where in some cases the cruise segment may not even exist. Erzberger and Lee[72,73] in 1980 published a FORTRAN IV program, whose quick acceptance suggests that these needs are now being well met. Their method, with details furnished in Ref. 73, employs specific energy as the independent variable and range-to-go as the principal state. Under prescribed constraints on engine performance, the Hamiltonian approach yields optimal energy histories (and associated thrust programs) for the climb, steady cruise, and descent phases. At ranges less than 500 n.mi., however, the cruise may take place below optimal altitude or may disappear entirely, depending on what thrust limitations are imposed. The user is offered a choice of performance index through specification of the coefficients in a weighted sum of mission fuel and time. This sum can be closely related to direct operating cost.

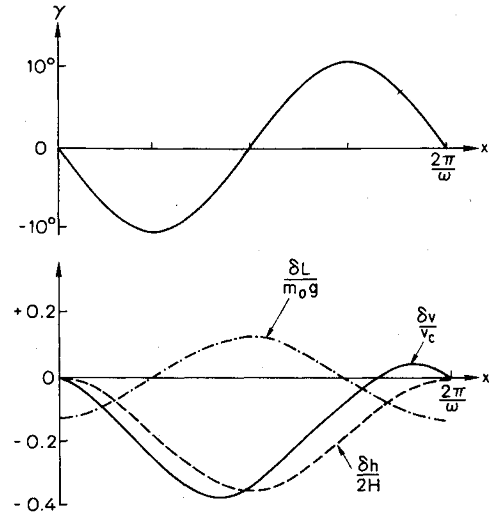

Fig. 8 One cycle of hypothetical reduced-fuel transport trajectory due to Breakwell and Shoaee (Ref. 71) plotted vs horizontal distance. Flight-path angle γ appears at top. In lower plot, perturbations of lift L, speed v, and altitude h are referred, respectively, to vehicle weight, steady-cruise v_c and scale height $H \cong 7.2$ km.

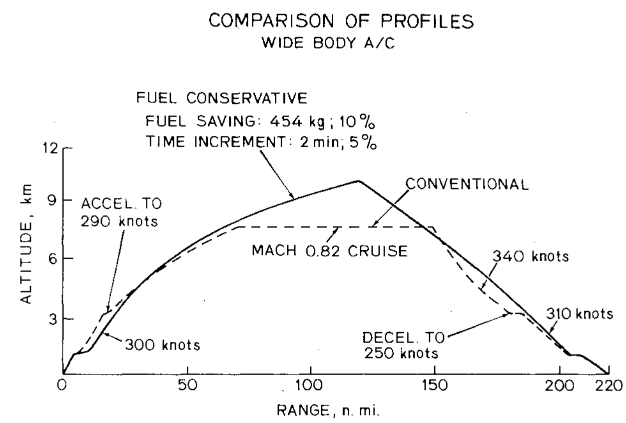

Fig. 9 Typical optimal flight path, computed by method of Ref. 73 for DC-10 transport. Note 10% fuel saving at a loss of only 2 min compared to more conventional operation (dashed).

Erzberger and Lee's first theoretical applications were to the DC-10 and Boeing 727-100 transports. In representative DC-10 applications over a 200-n.mi. route the minimum-fuel trajectory saved 7-10% compared with previous practice. Figure 9 was supplied by the authors and illustrates one of these cases. The conventional and optimal trajectories were both tried out on a piloted simulator; they are acceptable to passengers and involve no unreasonable demands on the propulsion system, crew, or ATC.

Schultz and Zagalsky[74] and Calise[75] are other recent contributors to complete-mission flight planning. Although no mechanization embodying the Refs. 72-75 concepts is yet in operation, Burrows[76] reports that they are to be implemented in the Flight Management System under development by Honeywell, Inc., for the Boeing 767. After supplying meteorological data, the crew will be able to generate optimal plans on a "real time" basis and, if desired, engage the autopilot to fly the chosen trajectory.

Reference 76 also examines the influence of atmospheric turbulence and describes the simulation of a small-amplitude "cyclic cruise." The latter is of the sort discussed in Refs. 70 and 71 but believed to be more feasible. For a 727-200 the

¶Note, however, that transonic drag rise is omitted. The predicted savings *may* disappear entirely in the face of greater realism.

study produces one instance of additional cost saving, over an optimum with a steady-cruise segment, of 0.4% resulting from an rms altitude variation of only 180 m due to stepwise throttle shifts every 25 s.

It is the field of dynamic performance, where high ratios of thrust to weight and extreme maneuverability permit rapid changes of or interchanges between kinetic energy and potential energy of height, that promises to provide the greatest opportunities for "getting the most" out of vehicles in the atmosphere. Under the leadership of Leitmann, Bryson, Kelley, and other dedicated theoreticians, this area of optimization research has been one of the most active of all since the 1950's. The literature is full of interesting examples, such as the supersonic interceptor and transport trajectories presented in the survey by Falco and Kelley.[77]

In his quest for realism the author soon settled upon the "F-4 minimum-time climb" of Bryson and Denham[78] as an initial illustration. Using realistic thrust and aerodynamic data, they applied the steepest-ascent method to a four-state, point-mass approximation. Angle of attack was the only control variable; full throttle was specified at all times. Figure 10 illustrates the flight path from Ref. 78 which formed the basis for a confirmatory trial by a Navy test pilot. It is intended to achieve minimum time from a horizontal start at Mach 0.38 and sea level to a horizontally constrained terminal condition of Mach 1.0 at 20 km. The "downhill" intermediate portion of the trajectory from Mach 1.0 to 1.6 is understandable because of high transonic drag and the engine thrust characteristics, but such a maneuver nevertheless contrasted sharply with previous practice.

The computed time in Fig. 10 is 332 s. When a F-4A was first flown according to the schedule of Mach number vs altitude (supplied on one small card), the climb was accomplished in 330 s. This is said to have represented a surprising improvement over previous attempts.

Subsequently, optimization had a continuing influence on the operation of F-4's and other fighters.**Thus Landgraf[79] summarizes the results of a series of time-to-climb records by stripped-down F-4B's during the spring of 1962, for which the trajectories were developed by a program due to Hague.[80] Final altitudes up to 30 km were attempted under less-stringent initial and final conditions than Ref. 78. The time to 20 km was thereby reduced below 200 s. Hague[81] remarks that the code used in Ref. 80 was later employed in studies on spacecraft orbits, maneuvering re-entry vehicles, cruise missiles, supersonic transports, etc.

Both simplifications of the problem formulation and more powerful mathematical tools have been added to trajectory optimization throughout the 1970's, far too rapidly for comprehensive citation here. Noteworthy in the latter category are the concepts of singular arcs, singular perturbations, and matched asymptotic expansions. As regards the former, the energy state approximation, which was first proposed by Rutowski[82] in 1954, has repeatedly demonstrated its ability (e.g., Ref. 83) to facilitate the calculation of practically important flight paths.

As Leitmann's 1962 survey[84] reminds one, the use of optimization on rockets and their trajectories is quite an old discipline of aerospace engineering. Perhaps the earliest, and surely a classic paper, is Malina and Summerfield's.[85] Therein the optimal masses for the stages of a multistage booster, intended to achieve a specified velocity increment for a given payload in field-free space, are determined by discrete-variable minimization. The performance index is total vehicle mass; structural-weight fractions and engine specific impulses are given for each stage. It is no accident that the relative proportions of most modern space boosters conform rather well to the recipes of Ref. 85.

**The F-4 series of fighters have ratios of maximum sea-level thrust to gross weight T/W ranging around 0.6. One can infer that trajectory optimization is, therefore, more important to best utilization of those lower-powered airplanes than, say, the F-15 with its $T/W \cong 1.1$.

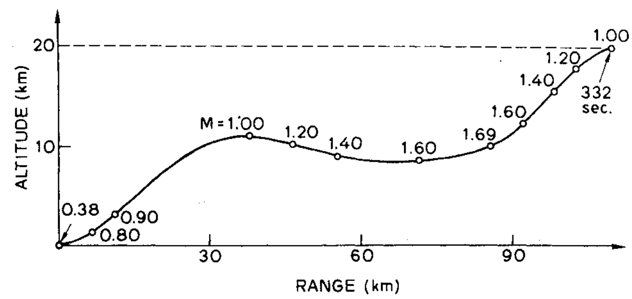

Fig. 10 Minimum-time flight path for F-4 fighter from sea level to 20 km, due to Bryson and Denham (Ref. 78).

When one explores the operation of ballistic missiles and boosters, it seems clear that the formal approach has had some influence, but appropriate documentation of actual cases is sparse. An exception is the Atlas-Centaur vehicle. Brusch[86] describes Convair's procedures, which involve a highly-constrained parametric optimization scheme based on the method of multipliers. A detailed example of its application for launching the FLTSATCOM communications satellite is given by Tramonti.[87] He explains how the program CONOPT effectively maximizes the spacecraft on-orbit weight under numerous conditions associated with safe operation of the launcher from Eastern Test Range to geosynchronous orbit. The author was informed[88] that all Atlas-Centaur trajectories are now optimized. The same may also be true of current Fleet Ballistic Missiles and of USAF ICBM's.

By contrast the first three booster families used in the U.S. manned space program followed simple gravity-turn trajectories, with allowance in the control logic for alleviating loads due to wind shears and turbulence.[89] The Space Shuttle ascent has, however, been the subject of formal analyses.[90,91] Again without documented confirmation, it is reported that the work of Ref. 91 played a part in the design of flight paths that will be employed during Shuttle operations.

Air-to-Air Combat and Differential Games

The theory of differential games (Isaacs[92]; Chap. 9 of Ref. 3) seems eminently suitable for such aeronautical applications as the engagement between opposing fighter aircraft. Indeed, under various approximations it has formed the background for numerous studies, some of whose solutions possess extraordinary richness. Thus Merz and Hague[93] (see also Ref. 94) study two vehicles restricted to a horizontal plane and turning at constant airspeeds. They calculate by optimal strategies the relative initial configurations from which one or the other can be guaranteed a "win," etc. As explained to the author by Tonkinson,[95] however, the one-on-one encounter so oversimplifies the nature of actual aerial warfare that its analysis cannot form an adequate basis either for the evolution of tactics or the design of new fighters. There have been efforts (e.g., Hague,[96] which has limited distribution) to determine optimal strategies for engagements involving several combatants, but their introduction into operations is believed to be a matter for the future.

On the other hand, there is a rather different route whereby the concept of the best has affected fighter development in the United States. Experience led to the identification of certain standard maneuvers whose outcomes, when they are performed optimally or suboptimally by a *single* airplane, furnish a measure of combat effectiveness. The key idea is to achieve a peak rate of swinging the velocity vector without unacceptable losses in total energy. One indicator of energy-maintenance capability is Boyd's[97] "specific excess power" (see, e.g., discussions in Refs. 97-99),

$$P_s = [(T-D)/W]V \qquad \text{(P1)}$$

where W is the weight, T the available thrust, and D the drag at a prescribed load factor or C_L. Taken from an illuminating

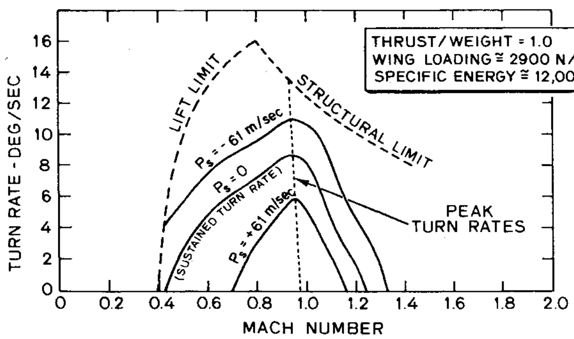

Fig. 11 Turn performance constant specific energy.

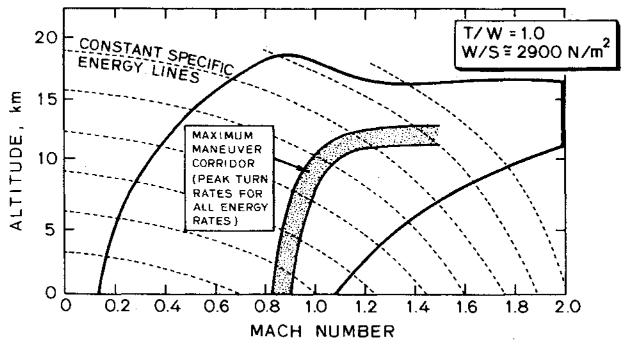

Fig. 12 Maximum maneuver corridor.

report by Patierno et al.,[98] Fig. 11 plots turn rate vs Mach number for a "typical fighter" under the circumstances depicted.

One first observes in the figure that possible rates are bounded on the left by a curve associated with $C_{L_{max}}$ and on the right by a limit load factor (7.33 *g* for most USAF fighters). These boundaries intersect where maximum angular rate can be achieved for a given altitude. This is the "corner velocity"—a point which was emphasized in pioneering research by Air Force officers (see, e.g., Ref. 100). Equally significant are the turns in Fig. 11 constrained by the requirement that P_s be held to a positive or negative constant value. Note that P_s is traditionally associated with rate of climb and that static best-climb profiles pass through its maxima for 1-*g* flight. Here, by contrast, it serves to estimate the gain or loss of kinetic-plus-potential energy.

From the peaks of the constant-P_s curves, Boyd defined a region in altitude-Mach number space wherein the constrained turn rates have their largest values; corresponding to Fig. 11, Fig. 12 shows such a region. There is considerable evidence that dogfights tend to migrate into these "maximum maneuver" corridors. Moreover, the capability of one air-superiority fighter over another can be judged both by relative P_s maps and by the locations and sizes of their corridors.

Optimization comes into play in the study of Ref. 98. There, a series of "yardstick maneuvers" were computed by an elaborate version of a steepest-ascent program like that of Ref. 78. With performance index chosen to be the minimum time to bring the velocity vector through a prescribed rotation (e.g., a 360-deg turn), the maneuvers were analyzed under various constraints on P_s and on the final state. For preliminary-design airplanes resembling the F-17/F-18 family, the optimal results usually come quite close to those found from Boyd's maximum-maneuver hypothesis. Both prove markedly superior to constant-altitude turns and other nonoptimal trajectories.

A second situation where differential games can influence military tactics involves the encounter between high-performance manned aircraft and ground-to-air or air-to-air guided missiles. In these instances the one-on-one idealization seems well justified. An early case of this kind of application is Vachino et al.,[101] who analyzed engagements between fighters and beam-riding missiles. Their formulation assumes constant-speed maneuvering in a single plane by the adversaries. The objective for the pilot is to maximize the closest distance of approach. Although the paper makes no such identification, this example resembles fighter intercepts by the SA-2 during the Vietnamese War. It is improbable that such optimal results were used for anything more than validation, but the tactic pictured in Fig. 4 of Ref. 101 is not unlike one that proved successful in the air battle.

More recently very sophisticated theory has been employed for the pursuit-evasion problem of an air-to-air missile attacking a fighter. Shinar and Gutman[102] treat the aircraft pursued by a faster missile as a linear differential game in three dimensions, with constant-speed point masses and with gravity neglected (on the grounds that the maneuver accelerations are high). The important issue of imperfect information available to the target aircraft is considered. Calise[103] also examines missile pursuit with emphasis on optimal control of the thrust magnitude. His powerful mathematical apparatus includes multiple time scaling and singular perturbations. As in other such studies, however, certain difficult-to-avoid simplifications limit the potential as a practical guide to tactics. In Ref. 103 target velocity is taken as a constant vector; it is pointed out that turns can be accounted for by up-dating the target state when recalculating the proportional-navigation guidance law at intervals.

Active Guidance and Control

The author remembers clearly a visit about 1948 to the laboratory of Y.T. Li at Massachusetts Institute of Technology, where Li demonstrated a simple optimal controller driving a rolling follower to the highest point of a two-dimensional track (see Draper and Li[104] for extensive theory and a piston-engine application). The track's shape could be adjusted, but the follower would always seek out and "hunt" about the peak. Whether this is the first example of its kind is not known. It was certainly one pioneer in the development of an enormously fruitful technology, and its builders introduced the adjective "optimalizing."

Bryson and Ho[3] remains today the source text for optimal control, and in particular its Chaps. 4, 5, 8, and 12-14 contain the foundations for thousands of hypothetical and practical applications which have appeared since about 1970. Throughout this period one finds many instances of a sort of competition between traditional methods and "modern" control theory. But where flying hardware and software are concerned the former have prevailed in almost every case.

The formulation of the optimal approach which underlies most studies contains a homogeneous, quadratic performance index associated with linear (or linearized) equations in state-vector form. It is often characterized as linear quadratic synthesis (LQS). Typical but by no means unique examples of its use include Berman[105] for the Grumman FSW entry and other advanced designs; Blight and Gangsaas[106] for landing approach of the Boeing Quiet Short-haul Research Aircraft (QSRA); and Bryson and Hall[107] for several simplified autopilots. Other cases of optimization by somewhat different methodologies are Tabak et al.,[108] on "multiobjective" control of fighter and lifting-entry types, as well as Michael and Farrar[109] on "suboptimal" feedback for a helicopter.

Although they fall outside the present scope, spacecraft have formed a subject of extensive research and occasional implementation. A noteworthy early example is Widnall's[110] minimum-time thrust controller for approach and touchdown of the Apollo Lunar Excursion Module. Reference 110 furnishes data from the first flight test in Earth orbit aboard Apollo 9; of course, the system performed outstandingly on subsequent lunar missions.

An important special case of LQS is the Kalman filter (Ref. 111 and Chap. 12, Ref. 3), a device for state estimation with maximum likelihood in the presence of disturbances assumed to have certain idealized statistical properties. A history of the

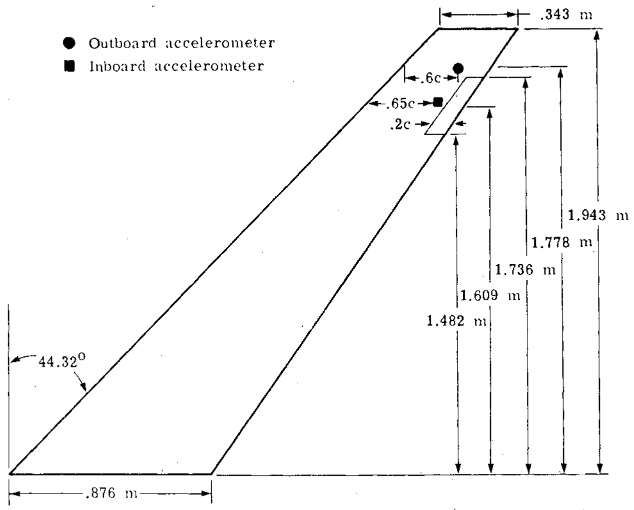

Fig. 13 **Plan view of flexible, transport-type cantilever wing model used by Newsom et al. (Ref. 115) to test control systems for flutter suppression.**

filter's role in both astronautics and aeronautics, with emphasis on his own contributions, was recently published by Schmidt.[112] Optimal filtering and prediction constitute such a uniquely advantageous approach that they have been incorporated into the navigation systems of several aircraft and spacecraft. Reference 112 mentions the first transport application, which is as part of the Northrop-developed system[113] now flying on the C-5A.

During early mechanizations of the Kalman filter it was discovered that divergent behavior can occur when process noise does not disturb certain combinations of states. Reinitialization at short intervals turned out to be one way of overcoming this difficulty, and lately various suboptimal schemes have been devised for making the realization more robust (Bryson[114]). As a result, most of today's inertial navigators routinely use the technology.

If "flight" in the wind tunnel may be included within the ground rules of this survey, then the tests reported by Newsom et al.[115] contain an excellent example of a control system synthesized by optimization. Their plant consisted of a flexible half-wing model (Fig. 13), mounted from a side wall of the Transonic Dynamics Tunnel at NASA Langley Research Center. This was a full-scale simulation of the structure and portions of the controls later to be flown in the DAST drone program (see Ref. 116), but the goal of increased flutter speed in free air was accomplished on the latter vehicle with more conventional stability-augmentation logic. The motion sensors are the two vertical accelerometers shown as black dots in the figure. The only "effector" available to influence aeroelastic stability is the small trailing-edge flap near the wingtip, but it has a powerful miniature hydraulic actuator with good frequency response in the 5-40 Hz range needed for interaction with significant modes of vibration.

After open-loop stability tests, two control algorithms were tried out on the model. The optimal version was designed in a rather unconventional way (Newsom[117]) since they chose not to employ a full-state estimator. LQS was first used to calculate an open-loop transfer function (e.g., between vertical acceleration and control-surface rate) which minimizes control activity in turbulence at the design point of dynamic pressure 7.72 kPa and Mach 0.9. Then the coefficients of a practical transfer function, in the form of a ratio of polynomials in frequency, were calculated by Davidon-Fletcher-Powell[118] so as to minimize the squared error, at several frequencies, between the desired and actual transfer functions. This synthesis, which is described in detail in Refs. 115 and 117, resulted in a system that increased the critical speed substantially at other experimental conditions beside the nominal. Figure 14 typifies the data obtained and indicates that the objective of increasing flutter dynamic pressure by 44% at Mach 0.9 was probably achieved. It is of interest that a different control based on the "aerodynamic energy" concept of Nissim[119] performed roughly as well as the optimal in this particular case.

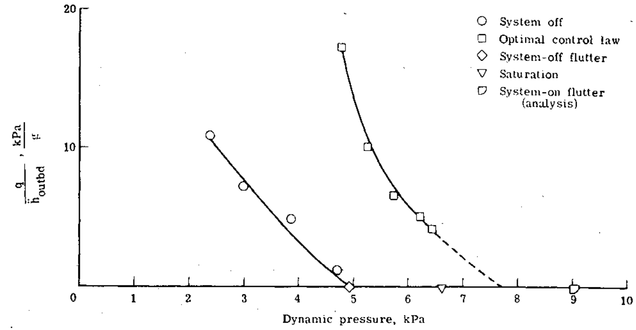

Fig. 14 Ratio of dynamic pressure q to amplitude of $\ddot{h}$ at outboard accelerometer (a measure of "aeroelastic damping" in wing of Fig. 13), plotted vs wind-tunnel dynamic pressure. Results for open-loop wing are compared with those achieved by the optimal controller.

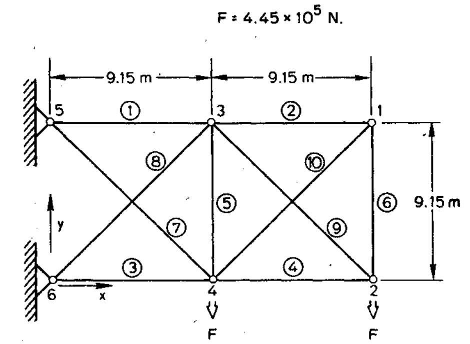

Fig. 15 Dimensions and numbering of elements in ten-bar frame example devised by Venkayya (Ref. 123).

Static Structural Synthesis

Turning to structures of aeronautical type, one notes that the earliest studies involved simple configurations of minimum mass to support one or two static loadings. Constraints were placed on strength, member properties, and possibly stability or nodal displacements. The 14-element frame employed for illustrative purposes in a preceding section is typical of these, except that joint locations are a fairly recent innovation in design-variable selection. The developers of optimizing codes have cut their teeth on a collection of such "test problems." Certain trusses with 3, 4, 10, 22, 25, and more bars show up repeatedly in their work (see, e.g., Refs. 120-122 for correlative data).

Several points about practical structural optimization can be made by reference to one of these: a redundant, ten-bar frame due to Venkayya,[123] whose dimensions and loading are shown in Fig. 15. This sort of configuration is selected because, unlike wing boxes and other built-up structures, the results of various analyses are insensitive to the details of finite-element modeling and can therefore be intercompared with confidence.†† The frame is made of a low-strength aluminum alloy. Minimum permissible cross-sectional area is 6.5 cm^2, and vertical displacements at the nodes 1-4 are limited to about ±5 cm. Figure 16 from Ref. 120 reproduces the history of five searches for the minimum mass. Unfortunately, space limitations preclude summaries here of

††Since no constraint is placed on cross-sectional shape in these designs, stability is not an issue.

information—extremely significant to the original researchers—regarding methods of analysis and optimization routines. Nevertheless, each solution is believed to be satisfactorily converged at the point where its plot is terminated. The curve marked "Venkayya" is the initial Ref. 123 design, whereas those labeled ACCESS-1 involve two different searches associated with the first version of this well-known synthesis code due to Schmit and various collaborators.

It is worth noting that programs exist today which converge much more efficiently than the best shown on Fig. 16. The numbers of iterations required there range, effectively, from 13 to 26. The amounts of computation per cycle were also quite variable. Another characteristic of the different approaches is that they generate final designs whose individual members may scatter somewhat in cross-sectional areas. Thus Table 24 of Ref. 120 shows two bars at minimum gauge in the Venkayya truss; two quite distinct ones in the Gellatly truss; one and four, respectively, in the designs due to CONMIN and NEWSUMT. On the other hand, the heavily loaded bars (Nos. 1, 3, 4, 8, and 9 in Fig. 15) turn out more nearly identical to one another across the board. Perhaps most important is that the minimum total masses agree remarkably; their range, from 2303 to 2319 kg among the six designs, constitutes less than a 1% variation.

The conclusion from this last statistic—not a surprise—recognizes that performance indices change only slowly in the vicinity of an extremal point. Once near to a useful result, the optimizer need waste little effort in squeezing out the last modicum of perfection.

The list of published examples like the foregoing, any one of which possesses some intrinsic interest and demonstrates some feature of the process, is endless. By contrast, circumstances where truly optimal products are part of the structure of U.S. operating aircraft are hard to find and harder to document. Lockheed's original C-5A proposal does contain a few pages[124] where the main fuselage is proportioned for minimum weight. The author is assured[125] that a simple but definitely formal search procedure lay behind the choice of skin thickness, the spacing of rings and longitudinal stringers, and the detailing of ring and stiffener geometry. Important stresses were those due to internal pressure, bending moment and shear. Damage tolerance of the pressure vessel was also a consideration. Figure 17, adapted from Ref. 124, shows four cross sections which define the optimum configuration. With minor modifications, it represents the production C-5A.

Figure 18 demonstrates how the spacing of C-5A fuselage rings affects quantities proportional to overall structural mass per unit of axial load P and of shear flow Q. A series of analyses like this one were assembled and led to an optimum spacing of 51 cm and other dimensions given in Fig. 17. Reference 126 furnishes background information on the methodology. It is stated[125] that similar studies underlie the sizing of wing, empennage, and fuselage panels for such Lockheed aircraft as L-1011, C-141B, and the proposed C-X.

As a last illustration of static structural synthesis at an intermediate level of complexity, consider the rectangular, cantilever wing box (Fig. 19) due to Schmit and Mehrinfar.[127] Two successive loading conditions are employed, at P_l only and at P_2 only. Constraints include strength, panel buckling, and maximum vertical displacements at the nodes. The use of filamentary composite material is simulated by means of layered panels in the skins of OLSR (orthotropic linear stress rectangular) type, whose fiber orientations are prescribed. The 28 total design variables (allowing for top-to-bottom symmetry) consist of six stiffeners (TRUSS), nine shear webs (SSP), and twelve skins (OLSR). The clever and efficient scheme of Ref. 127 avoids dealing with 28 independent

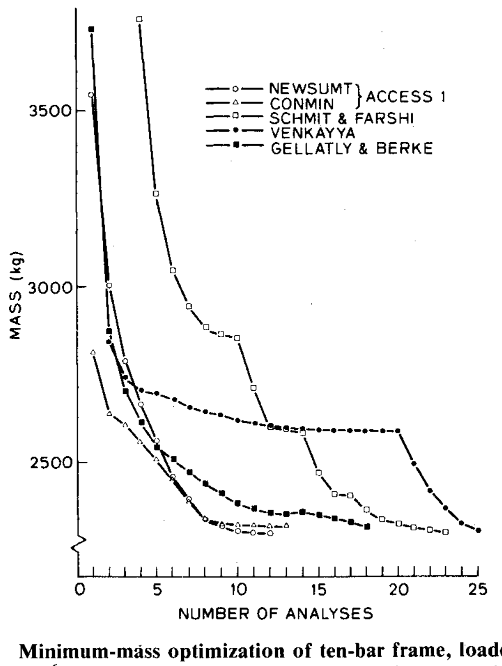

Fig. 16 Minimum-mass optimization of ten-bar frame, loaded with $F = 4.45 \times 10^6$ N. (See Ref. 20 for details and literature citations. Material is typical aluminum alloy.)

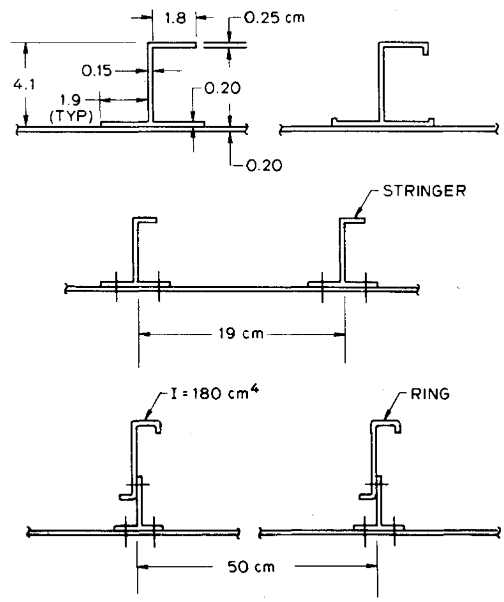

Fig. 17 Four cross sections describing optimized structure of C-5A fuselage skin.

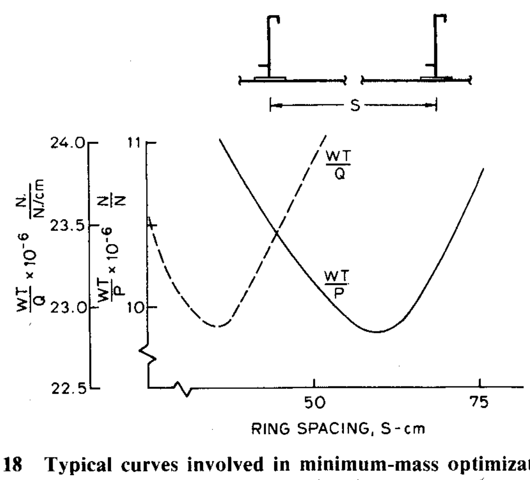

Fig. 18 Typical curves involved in minimum-mass optimization of C-5A fuselage. Shown are variations with ring spacing of normalized weight per unit of axial load P and shear flow Q.

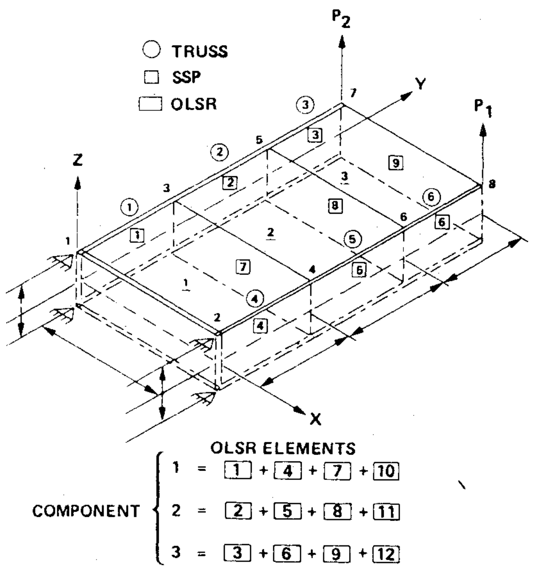

Fig. 19 Defining stiffeners, shear webs and skin elements in cantilever wing box optimized by Schmit and Mehrinfar (Ref. 127). Loads were separately applied at P_1 and P_2.

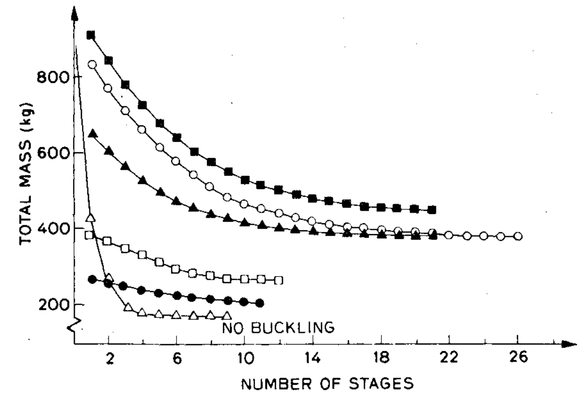

Fig. 20 Minimum-mass optimizations under various design constraints of Fig. 19 box. (Open triangles are a case in which buckling was ignored. See Ref. 127 for further details.)

quantities by a multilevel optimization. At the system level, ACCESS-1 seeks minimum mass while making approximations to active constraints and "linking" associated variables. The analysis is passed cyclically between the system level and more exact calculations at uncoupled component levels which minimize changes in component stiffness. The actual search procedures in both stages are based on the quadratic extended interior penalty function, leading to a sequence of unconstrained minimizations (NEWSUMT, Refs. 128 and 129).

For loads $P_1 = 33{,}300$ N and $P_2 = 66{,}500$ N applied to a wing of span 4.82 m and chord 2.54 m, Fig. 20 reproduces some mass iteration histories from Ref. 127. All cases start from conservative, feasible designs and appear converged before 20 cycles. The lightest and most rapid is unrealistic, since no panel buckling is accounted for. The others represent various combinations of buckling constraints and allowable sandwich-core depths in the upper and lower skins.

It is mentioned that most investigations of the sort referenced here pay no attention to a question which bears on the utility and acceptability of their products: what are the sensitivities of the performance index and element sizes to altering such system parameters as allowable strength, deflection, geometrical details, and the like? Until recently, sensitivities had to be computed by a laborious process of perturbing the quantity of interest and reoptimizing. A 1981 paper by Sobieski and Barthelemy[130] has, however, demonstrated how the required derivatives may be found in a more direct and efficient fashion.

Realistic Structures with Dynamic or Aeroelastic Constraints

The author's own mild affair with optimization centered upon one-dimensional structures constrained to vibrate, flutter, or diverge at or above prescribed thresholds on the critical eigenvalues. With several doctoral students and other colleagues, he published on systems with both distributed parameters (Refs. 131-133) and discrete design variables (e.g., Ref. 134). Some of the results were intrinsically rewarding, but most were a long way from the complicated, built-up lifting surfaces of real aircraft with their multiple design criteria and tight constraints.

With regard to the search for such practical applications, an unexpected discovery is that two of the earliest occurred where avoidance of flutter dominated other considerations of strength and stiffness. The first of these was carried out under the supervision of the late S.J. Loring at Vought-Sikorsky Aircraft in 1941. It involved increasing wing stiffness in the F 4U-1 Navy fighter so as to bring the critical speed to an acceptable level.

As abstracted from information furnished by M.J. Turner, Boeing Commercial Airplane Co., the story is as follows:

> The outer panels of the inverted gull wings were single spar structures, hinged at the root, with the spar located at about 30% chord from the leading edge. Torsional loads were carried by the single-cell torque box forward of the spar, and fabric covering was employed on wing surfaces aft of the spar. 0.50-caliber machine guns (3 per side) and ammunition boxes were supported, behind the wing spar, by outer panel wing ribs. Provision of considerable rigidity in these ribs, and backup structures forward of the wing spar, was required to prevent unfavorable dynamic interaction of wing torsion and rib flexure during flutter.
>
> Severely constrained by available computing facilities (Marchant desk calculators), we modeled the outer wing structure aft of the spar as an assemblage of independent cantilevers with flexible root restraint, each carrying a share of the mass of armament, control surfaces, etc. Each cantilever and its backup structure was represented as a statically determinate, finite-element assemblage (coarsely subdivided) of rods and shear webs.
>
> Assuming that structural mass is small relative to supported mass, Loring had shown that, for a given value of natural frequency in the fundamental mode of a statically determinate structure, the structural mass is minimal when amplitude of axial stress, σ, and shear stress, τ, are constant and $\sigma/\sqrt{E} = \tau/\sqrt{G}$ (constant strain energy density) during vibration at constant amplitude in the fundamental mode [see Turner, Ref. 135]. Rib structures were sized by this criterion for a prescribed fundamental bending frequency of 33 Hz. In addition to providing sufficient stiffness to satisfy wing flutter requirements this design procedure also provided beneficial support rigidity for the armament.

The all-movable horizontal stabilizer of the B-1 bomber is a more recent instance where considerable material had to be added in order to ensure acceptable flutter performance. As described by Siegel,[136] resizing was carried out by an aeroelastic optimization program STROP. Its basic formula, which is an approximation but one that often yields results quite close to optimal, required nearly uniform strain energy density throughout the stiffness-effective structure when

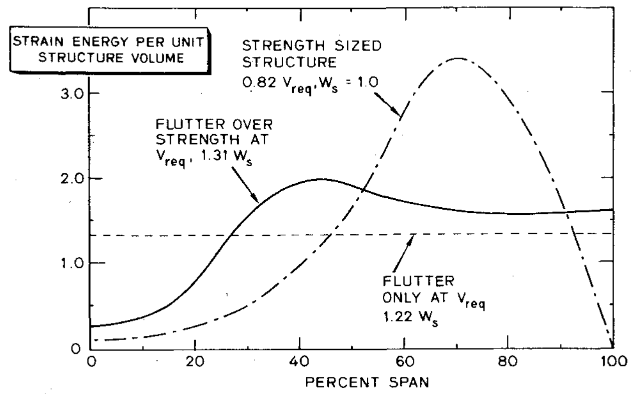

Fig. 21 Spanwise distribution of specific strain energy (dimensionless) in the flutter mode for three designs of B-1 horizontal stabilizer (Ref. 136). The solid curve is a minimum-mass case, which meets design criteria for both strength and flutter.

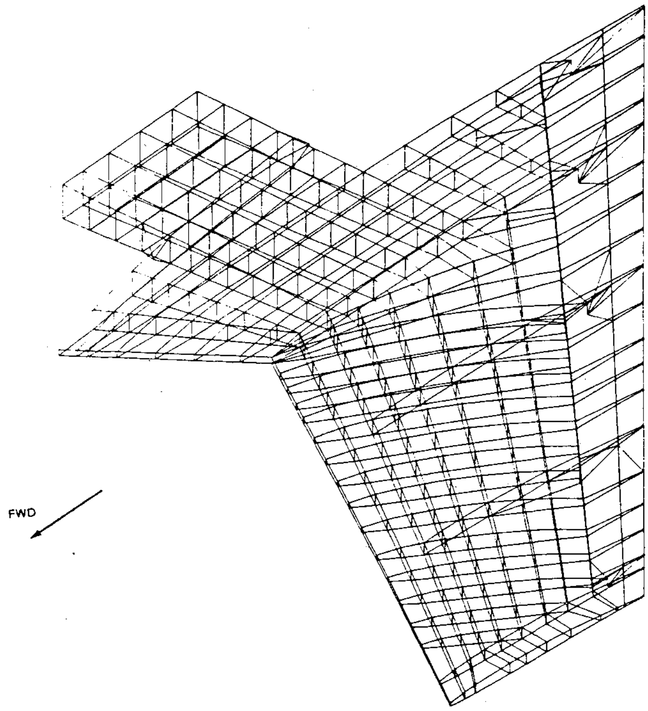

Fig. 22 Representation with approximately 2400 finite elements of FSW fighter wing designed by Grumman Aerospace Corp. (unpublished). This structure is reported to be optimized for divergence and strength by programs FASTOP and ASOP-3.

deformed in the flutter mode. Flexible model tests during early development phases showed that a conventionally designed B-1 tail was deficient by almost one-half in its critical dynamic pressure at transonic speeds.

In practice STROP began with a beam-rod idealization of the tail with material adequate only for strength. After estimation of flutter speed by modified strip theory and coupled normal modes, positive increments were made to skin thickness at those stations where specific strain energy was the highest. Flutter was recalculated and thickness revised iteratively until the required margin of safety on speed was attained. Figure 21, adapted from Ref. 136, plots dimensionless strain energy vs spanwise distance for three cases. The solid curve relates to a near-optimal revision of the original design (dashed-dot), whereby flutter dynamic pressure was increased 33%. The paper points out that a 31% addition to total mass was needed, as compared to 49% for a satisfactory but nonoptimal candidate. For the latter, strength-designed skin thicknesses were simply increased proportionately until the requirement on flutter was met.

Several other more elaborate schemes for sizing lifting surfaces with a predominant flutter condition have appeared (see, e.g., Ref. 17) since the 1960's. The intuitive concept of changing the mass m_i of each stiffness-effective member in direct proportion to its influence on flutter speed V_F, as measured by the derivative $\partial V_F/\partial m_i$, is reported[137] to have been employed at Lockheed-California Co. for twenty years. Reference 138 expresses this criterion as follows for a case where V_F itself is acceptable and therefore to be held constant:

$$\{\Delta m_i\} \sim \left\{\frac{\partial V_F}{\partial m_i}\right\} - \left(\frac{\lfloor \partial V_F/\partial m_i \rfloor \{\partial V_F/\partial m_i\}}{\lfloor \partial V_F/\partial m_i \rfloor \{1\}}\right)\{1\} \qquad \text{(F1)}$$

References 139 and 140 assess five flutter-related computational approaches and explain their relationships to the Eq. (F1) concept of "adding material where it does the most good."

Regrettably, the only large published example of application[140] concerns an arrow-wing supersonic transport which was never built. It should be mentioned that this transport structure contains a demandingly large number of design variables in its initial configuration, prior to variable linking and other means for basis reduction. It forms the subject of a more recent study by Green and Sobieski.[141]

One will observe that the mass increments called for by Eq. (F1) will vanish if the elements of the column matrix of derivatives $\partial V_F/\partial m_i$ are all equal to one another. This is a straightforward way of arriving at the basic criterion underlying the Grumman Aerospace Corp. flutter optimization program FASTOP.[142] Reference 142 and other papers describing the program actually develop this result in a context of more sophisticated means for deriving optimality criteria relevant to deflection, natural-frequency, and aeroelastic constraints. Nevertheless, constant $\partial V_F/\partial m_i$ turns out to provide a very efficient redesign algorithm. FASTOP has formed the tool for several interesting paper studies. For instance, Vol. I of Ref. 143 deals with the difficult problem of achieving adequate flutter margins by optimal mass-balancing on a fighter which must carry many different combinations of stores at the wing pylons.

Another important Grumman contribution to the armamentarium of practical optimization is the program ASOP, which finds the minimum mass of lifting surfaces and other built-up configurations. Finite-element representations with over 1000 nodes are accommodated, along with 100 or more independent design variables. The original ASOP employed only static constraints on strength and deflections for metallic structures. The latter were handled by requiring all derivatives of the constrained displacements with respect to element masses to be equal, whereas optimal strength was approximately attained by fully-stressed design—i.e., each element was maximally stressed in at least one loading condition.

Reference 144 describes ASOP-3, an expanded version encompassing filamentary composite members, whose reinforcement directions may have considerable freedom. In the last couple of years Grumman has combined the fully-stressed capability with aeroelastic constraints as embodied in FASTOP. The resulting program is especially useful for designing complicated structures with orthotropic skin elements. Although full publication of the applications is not yet possible, a major role is being played by these tools in developing composite lifting surfaces like the wing of a FSW fighter—one case where previous experience with metallics is of little value for choosing optimum ply orientations and where certain results prove highly counterintuitive. Wilkinson has supplied the author with Fig. 22 as an example of what can be handled. Approximately 2400 finite elements make up the wing and carry-through structure. Over outboard portions of the main structural box skins, up to 34 plies of graphite-epoxy reinforcement had to be added to a strength design.

These were needed to ensure freedom from the bending-torsion divergence phenomenon, which can be a severe threat to FSW aircraft.

Associated with the use of composites for wings and tails of high-performance vehicles is the idea of "aeroelastic tailoring," whereby the coupled bending and torsional deformations are put to work in a constructive way to avoid instabilities and/or to improve various aerodynamic properties. An early investigation which combined this idea with the optimal choice of ply orientations led to the program Wing Aeroelastic Synthesis Procedure (WASP) of McCullers and Lynch.[145] WASP involved 30 design variables: one orientation angle and nine coefficients of a polynomial thickness distribution for each of three symmetrical upper and lower layers of composite skin. With constraints on strength, divergence and flutter the skin mass was minimized by a Fletcher-Powell[118] type of search routine.

Renamed TSO (tailored structure optimization), a much extended 1976 revision of WASP is presented and applied to aeroelastic tailoring of the YF-16 and more advanced designs in Refs. 146 and 147. Certain of these constitute combined structural and aerodynamic optimizations, in the sense that constraints are enforced which cause favorable wing twisting at high maneuver load factors. The aim is a tendency toward elliptical spanwise lift variation and minimum induced drag. For example, Ref. 146 compares drag polars between an essentially rigid YF-16 and an optimally tailored composite replacement. For one flight condition at $C_L = 0.8$, C_D is reduced by more than 26% from the 1400 counts estimated for the baseline vehicle. Again, it is disappointing that no such optimal products have yet been incorporated[148] into any of this series of fighters.

HiMAT is the acronym for the highly-maneuverable aircraft technology demonstrator built by Rockwell International, two of which are now undergoing testing at NASA Dryden Flight Research Center.[149,150] HiMAT is the only vehicle airborne today which embodies aeroelastic tailoring. Approximately 95% of its surface is fabricated of graphite-epoxy skin, fabric or tape. A preliminary version of the TSO program was involved in early development stages of the wing and canard stabilizer. The final structural design was carried out in more conventional fashion, however, with analyses done by means of finite elements on NASTRAN.

The one respect in which HiMAT may be characterized as optimal (see Ref. 146) is the use of tailoring to minimize induced drag in rapid turning maneuvers. This vehicle was required to maintain at least zero specific excess power in an 8-g level turn at Mach 0.9 and 9-km altitude. Reference 150 explains how this goal was achieved by first using aerodynamic optimization to prescribe ideal spanwise lift distributions on the close-coupled tail and wing. The corresponding distributions of twist were impossible to reproduce exactly by means of aeroelastic deformation, but the surfaces were tailored and successfully approximate what is desired in flight.

A leading edge device was installed on the wing so as to ensure satisfactory twist in the cruising condition. Otherwise the specified cruise range might not have been met. Figures 10-14 of Ref. 150 indicate how well the actual structure matches the requirements on deformation under load.

Rockwell is also active in design studies on FSW fighters. Reference 151 and portions of Ref. 152 summarize, among other details, how TSO is being employed to achieve minimum wing mass under multiple constraints. Perhaps the most significant of these is to avoid the aforementioned problem of divergence. Figure 23, reproduced from Ellis,[151] shows the structural arrangement. With graphite-epoxy in the upper and lower skins, it proves necessary to orient the reinforcements in 70% of the plies at 9 deg ahead of the sweptforward structural axis, thus obtaining divergence speeds comparable to an unswept wing of the same mass. Figure 24 lists the cover masses and critical dynamic pressures

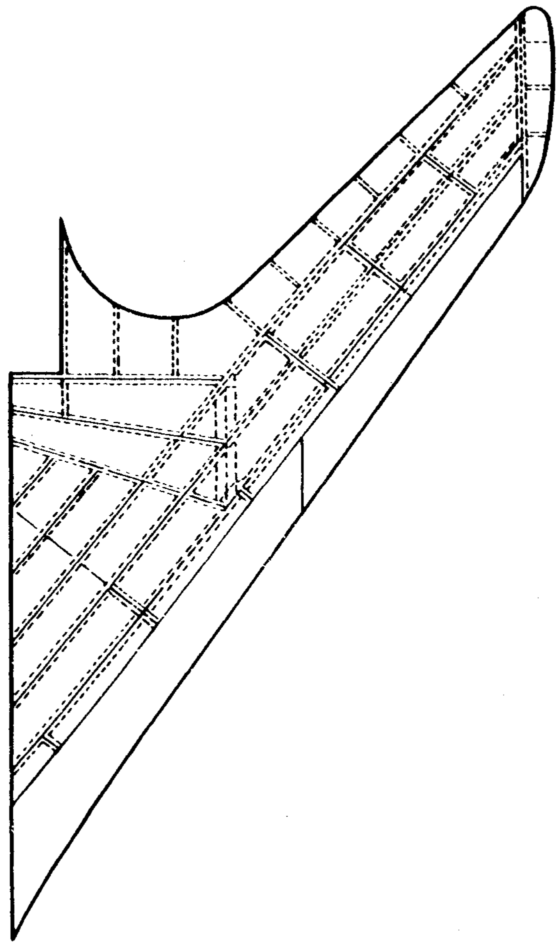

Fig. 23 Plan view (Ref. 152) of Rockwell International FSW wing design, showing locations of principal structural elements.

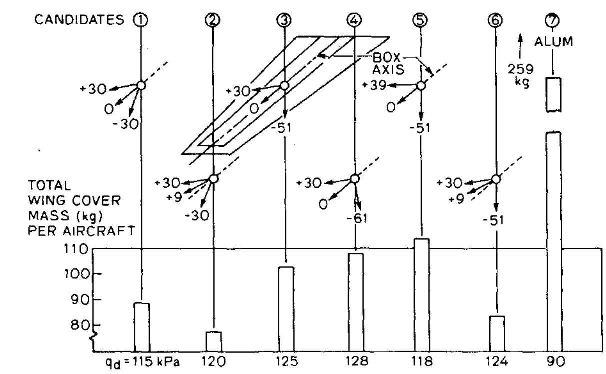

Fig. 24 Information on cover masses, divergence dynamic pressures q_d and skin ply orientations for seven candidate designs for Rockwell FSW fighter wings. (Adapted from Fig. 5, Ref. 151. Second from right is chosen design.)

q_D for seven candidate wings. As can be seen, an untailored aluminum structure is more than twice as heavy as the composite versions. The one actually selected for the proposed vehicle is second from the right, with q_D well in excess of the required value 95 kPa.

Summaries of yet a third series of FSW studies, by McDonnell-Douglas Corp., will be found in Triplett.[153]

It should be a source of chagrin for American aerospace engineers that minimum-mass optimization of complex primary structures has been employed routinely since 1976 in France at Avions Marcel Dassault-Bréguet Aviation. Petiau and Lecina[34] state, among other things, that the method described in their report "is applied systematically in all our

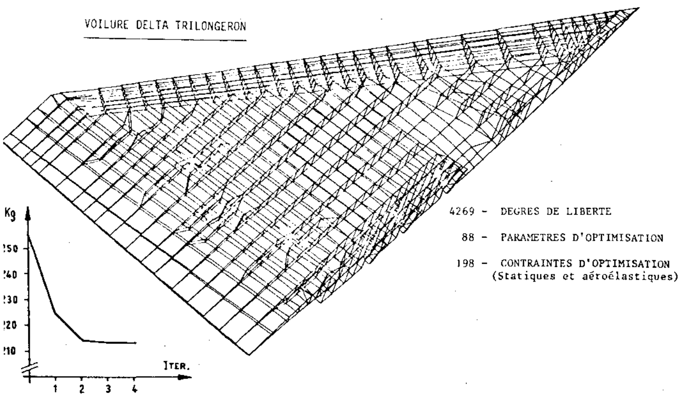

Fig. 25 Finite-element representation of delta fighter wing, whose optimization is reported by Petiau and Lecina (Ref. 34). Curve at lower left shows convergence to minimum mass after three iterations.

projects, after having been introduced on the Mirage 2000 and Mirage 4000" fighter models. The reader is directed to their paper for full details. Finite-element representations with order 5000 degrees of freedom can be optimized, constraints being available on stress, local buckling, displacement, static aeroelastic properties, vibration frequency, dynamic response, and flutter. As in the case of ACCESS and other current U.S. programs, Dassault employs extensive linking, works with inverses of the "natural" design variables, successively linearizes the constraints, and searches with a conjugate gradient scheme. Through extensive use of interactive graphics and the evolutionary application of existing finite-element capability, full acceptance by experienced structural designers is said to have been achieved. The overall cost for computing an optimal result is estimated at eight to twelve times that for one corresponding analysis of the same configuration.

Two sample applications are summarized in Ref. 34: a graphite-honeycomb empennage with 135 design variables and a three-spar delta wing. Since the latter is clearly taken from an operational Mirage, it is chosen as the present paper's most elaborate example. Figure 25 reproduces the finite-element idealization, which has 4269 degrees of freedom. There are 88 independent design variables, subjected to 198 static and aeroelastic constraints (minimum gauges included). Generally the most critical conditions were a pullout maneuver for the lower skins and a roll maneuver for the upper surface and fuselage attachments, although aeroelastic and other requirements were effective in certain areas.

Figure 25 also shows the very rapidly convergent search. Three iterations were required to proceed from a feasible starting point at 254 kg (for the half wing) to an extremum, 41 kg lighter. The authors of Ref. 34 comment that a transfer of material, during the process, toward the trailing edge and wing root is quite counterintuitive. They also point out that this highly redundant optimal structure differs considerably from what would have been obtained by the fully-stressed approximation.

Finally, two features of the Dassault methodology deserve mention. By interactive means, it is possible for the operator to examine sensitivities. He simply displays a table of derivatives, which show the effects on each constraint of small changes in each design variable. A recent modification to the basic program permits maximization of structural margins of safety, with a limit on total mass, to be substituted for the conventional mass minimization.

Complete Vehicle Configurations

Aircraft preliminary design (PD) comprises such a host of tradeoffs that one might have expected to see a more formal approach combined quite early with the traditional combination of experience, intuition, and sequential unidirectional searches of parameter space.‡‡ Indeed, Boeing's Automated Engineering and Scientific Optimization Program (AESOP) had been applied by 1968 to a variety of examples, including a hypersonic cruiser.[154] Thelander[155] published in 1967 a variational scheme whereby performance was optimized by selection of a set of variables related to the propulsion system, wing, and body geometry. As with the component technologies discussed above, however, it has not proved easy to connect studies like these to the vehicles actually manufactured by their authors' employers.

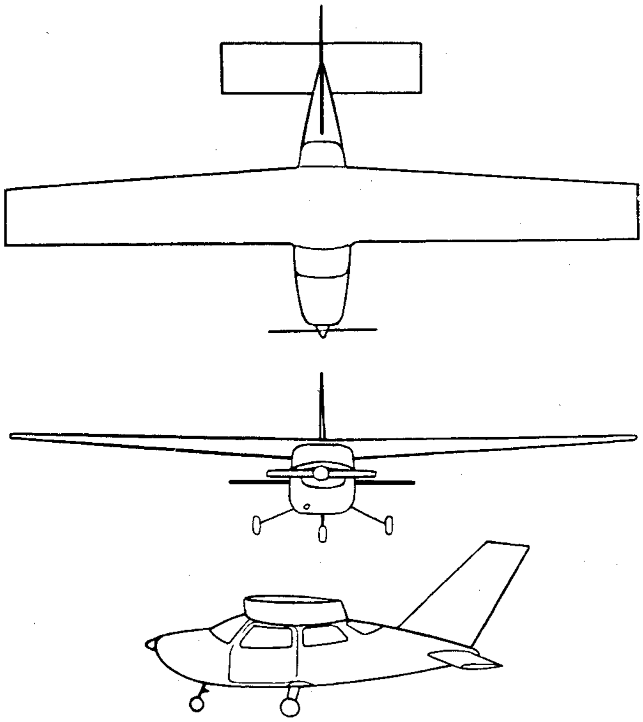

Fig. 26 Three views of single-engine light aircraft optimized for minimum cost of ownership (Ref. 160).

Despite this apparent reluctance on the part of potential users, the undismayed optimizers continue their efforts. Thus Ardema et al.[157] make a strong case for improved energy efficiency. Despite relatively conventional technology and a FAR-36 noise constraint, their best 1976 design theoretically achieves seat-miles per unit of fuel consumed 30% greater than contemporary wide-body transports. More recently Sliwa and collaborators (Ref. 158 is typical) show remarkable improvements possible for medium-range commercial carriers. Focusing on refinement of the structure, Ref. 159 demonstrates how the introduction of composites into an SST of given overall size can reduce structural mass from 99 to 77% of the (fixed) payload (see Refs. 138-140). Numerous other similar results could be cited.

In the interest of brevity, just two rather unique configuration studies will be illustrated—one because it was apparently the first light-airplane application, the second because it is going to "fly." In 1971, doctoral candidate B.W. Silver[160] completed an attempt to identify the best in single-engine personal aircraft. His performance index was the 10%-discounted total cost of ownership over a ten-year lifetime. Design variables were engine installed power, fuselage length, and five measures of wing size and proportions. A key requirement was that the vehicle carry a given payload a given number of miles per year. Constraints were variously placed on static stability, vertical tail volume, range, stall speed, and angle of climb.

Having collected extensive data on 1969-model light aircraft, Silver developed regression formulas which related the

‡‡Silver[156] prepared a useful literature survey up to 1969.

Table 2 Design variables in Ref. 161 transport study

Quantity	Range of variation
Takeoff gross weight, kg	136,000-362,000
Military payload/takeoff gross weight	0.15-0.45
Wing loading, kg/m^2	440-825
Sea-level static thrust/gross weight	0.17-0.27
Wing structural aspect ratio	7-15
Wing leading edge sweep angle, deg	10-45
Wing mean thickness ratio	0.09-0.15

Table 3 Typical design specifications

Military and commercial cargo versions of each configuration
Military version
Takeoff critical field length ≤ 2440 m
Sea-level, standard day, engine-out climb gradient ≥ 0.03
Range mission
Initial cruise altitude ≥ 8500 m
Outsized payload (90,000 kg)
MIL-C-5011A ground rules
Commercial version
Takeoff FAR field length ≤ 3040 m
Sea level, standard day
Provide cargo box cross section for two lanes
Gross payload to make same maximum zero fuel weight as military version

cost of ownership of each to various combinations of some 68 independent variables. A program AIDA II employed these data for improvements by means of a "learning-line" scheme, which undertakes random searches in design space based on an attenuated sum of prior successful moves.

Figure 26 depicts one converged redesign from Ref. 160, an "optimized" version of the 1969 Cessna 177 Cardinal. This case placed a value of $5/h on each passenger's time as an indirect means of forcing toward higher cruising speed. The baseline Cardinal's wing span of 11.7 m has moved up against a constraint of 13 m. In addition to the consequent increase of aspect ratio, there are an obvious reduction in tail length and increase in empennage area. In 1971 dollars, the ten-year discounted cost of the airplane of Fig. 26 is estimated to be $57,471. This represents a 7.4% reduction below the corresponding estimate for the baseline.§§

With regard to large and expensive aircraft, one can anticipate that rising fuel costs and other inflationary features will soon compel more attention to "mathematical PD." Greatest success may be expected from a blending of the enormous experience base now stored in computer data banks by the major manufacturers with the best of discrete search algorithms. Unconventional technologies like active control, variable-cycle engines, and composite materials will accelerate the trend. Even the skillful PD generalist is unprepared to combine them without help.

The most advanced effort to bring about this synthesis known to the author is the one summarized by Jensen et al. in Ref. 161. First presented in January 1979, their paper has the goal of optimizing a blended commercial and military cargo transport. Boeing's Airplane Responsive Engine Selection (ARES) data management system provides a framework of regression analysis for constructing second-order relations between configuration parameters, serving as independent design variables, and such performance indices as costs, fuel consumption, and gross weight for a prescribed payload and range. Table 2 lists the seven variables actually employed in Ref. 161; up to ten can be specified. Table 3 gives the important constraints—here the specifications as written by a military or airline customer.

The reader is referred to the source paper and references therein for complete information on the methodology, which is the outgrowth of research sponsored by the U.S. Air Force. New technologies include composites, supercritical-wing aerodynamics and energy-efficient engines. It must be mentioned that dynamic loading and aeroelastic phenomena are denied an explicit role in the analyses. Weight data have been validated against previous successful transports, however, and the paper cites active load alleviation and mode stabilization systems as ways of avoiding severe impacts.

Table 5 of Ref. 161 lists the principal characteristics of seven representative and comparable designs. These range from "traditional" airplanes with minimum gross weight or direct operating cost (DOC) to an unconventional minimum-fuel transport, whose aspect-ratio 14.2 wing and gross weight are surprisingly large. Not all details of these designs and their subsidiary penalties due to the various choices of performance index can be reproduced. Some further interesting results are as follows:

1) Designs optimized on gross weight, life-cycle cost (LCC), acquisition cost, flyaway cost, ratio of LCC to productivity, DOC, and fuel consumption differ by less than 9% in gross weight (220,000-248,000 kg). Other characteristics, such as engine thrust, wing sweep, wing area, and cruise Mach number show much greater variability.

2) Designs for minimum gross weight and DOC show the smallest penalties relative to optimum values of the other performance indices. On the other hand, the minimum-fuel transport seems less desirable because of low productivity and severe disadvantages in measures of cost.

3) By means of weighted combinations of performance indices, it is possible to construct "compromise" designs and examine tradeoffs among these indices. Some of the solutions produced by this technique appear to be superior to any of the single-index optima.

The author is informed[162] that Boeing's entry for the C-X transport development contract received its preliminary sizing by the methods of Ref. 161. Furthermore all PD activity in the military and commercial airplane companies beyond the Boeing 767 is expected to employ this new tool.

Concluding Remarks

There are better places far away, and at this place I am anyway.

German humorist, Moritz Busch, as quoted in *The Wind and Beyond*[177]

These conclusions must begin with an expression of regret that space limitations and the author's inadequate background have hindered suitable attention to work on optimization outside the U.S. Thus every researcher in the field is indebted to the fundamental contributions of such Soviet scientists as Pontryagin (e.g., Ref. 163). M. Dublin has supplied an annotated list of 44 significant Russian papers, dating from 1963 to the present. Reference 164 is typical of recent British efforts toward the practical automation of structural design. In 1980 Schneider et al.[165] described the software used to minimize aircraft structural mass by a major German manufacturer. References 13, 166, and 167 are selected randomly as further indications of interest throughout the world.

With the notable exception of Ref. 34, however, no foreign publication examined to date presents unequivocally any results embodied in an operational aircraft. Thus the evidence accumulates regarding the curious gulf between what might be and what is.

How can one account for what seems to be a paradox? It is certainly not true that *all* novel and complicated technologies

§§The reader is cautioned about the hypothetical and simplified nature of this study. In a recent conversation, Silver pointed out that, if comprehensive constraints had been included on dynamic stability and flying qualities, the airplane pictured in Fig. 26 might have been quite different. Thus the short tail was probably unrealistic, whereas the call for increased wingspan was a sound recommendation.

require decades to be adopted by the working aerospace community. Finite-element tools of structural *analysis,* for instance, grew up with the industry and were put to work nearly as fast as computational facilities permitted. Finite-difference codes for predicting nonlinear transonic flows were applied to such designs as HiMAT within a year or two of their availability.

Perhaps a contrapuntal listing of some considerations for and against the practical employment of optimization may help to explain the difficulty.

1) The literature bulges with typical gains in weight and cost that this approach can *theoretically* achieve. Yet design specifications do not require its use, its methods are often unfamiliar, one's competitors are not seen to have embraced it.

2) Computers become more efficient by the week, and the computational demands of optimizing codes have been dramatically reduced in the past few years. Yet these costs still exceed by one or two powers of ten those of comparable analyses. Trustworthy codes are not easy to come by, and they tend to be much harder to use in practice than even the largest analytical programs.

3) At the PD stage, optimization has great potential as a sound way of choosing among alternative concepts, all the way from overall configurations to propulsion systems and materials of construction. Later on in the development process, however, it may lead farther up blind alleys. A wise old engineer once asked what good it does "to optimize the hell out of a bad idea."

4) There are many situations, notably in airline operations, where a cost reduction of a few percent can mean many millions in increased profits. On the other hand, as already mentioned, optima tend to be very flat. The experienced designer or operator feels he can do just about as well as any computer code, and he thereby retains greater control of decisions.

5) In the absence of experience when new technology is being tried for the first time, the search for extremals can produce unanticipated, surprising and often very satisfactory discoveries. Yet the counterintuitive may also be counterproductive and even ridiculous. Very undesirable consequences can result from omission or careless handling of constraints. Factors like manufacturing intricacies and public prejudices may be overlooked or impossible to quantify.

6) When everything else is said, the "not invented here" factor may prevail over the most convincing advocacy.

Nevertheless, there are enough positive indications on the record to justify the author's expectation of impressive progress during the 1980's. In his lecture at AIAA's 50th Anniversary Celebration, Vanderplaats[168] observed that "the optimization algorithm is usually not a critical issue." In the area of his specialization he assured that "the technology now exists to efficiently design structures defined by several hundred design variables under multiple loading conditions" and almost any other conceivable constraint. It is a charming sign of maturity that Ref. 161 does not even bother to mention what scheme is employed to search for optimal transport configurations. Comparable hopeful statements could be made about any other class of application named in Table 1.

The objections about difficulty of use are likely soon to fade in the face of several current projects which will include efficient "distributable" optimization packages as part of familiar software collections like NASTRAN. Success stories like those reported in Ref. 34 are becoming common knowledge. One begins to wonder how much of Dassault's penetration of international military markets might be due to lighter-weight machines for a given performance. Finally, innovations like higher transonic L/D, new varieties of composite matrices and graphite reinforcements, variable-cycle engines, and FSW are challenging the designer's reliance on past dogma.

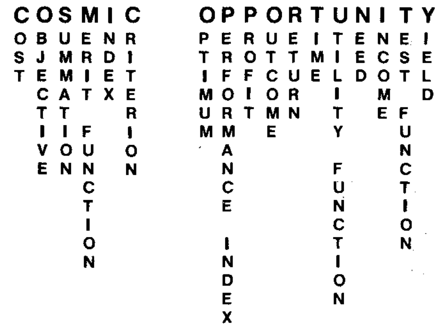

Fig. 27 The author's view of the future promise of formal optimization technology, constructed from initial letters of seventeen synonyms for the "performance index."

There are many names for the objective to be minimized, herein referred to as the "performance index." In Fig. 27, seventeen of these names are arranged so as to suggest—if allowance may be given for some hyperbole—one person's view of this technology's future promise for aerospace.

Appendix

Optimal Euler Columns

As an elementary illustration of minimum-mass structural design by search of function space, the simply supported column subject to requirements on Euler buckling load and compressive strength serves well. The results perform double duty when incorporated into the pin-jointed trusses of Figs. 3 and 4. This problem has a rich and interesting history. With regard to various types of buckling of columns having *uniform* cross section, no better survey can be cited than Hoff's 41st Wilbur Wright Memorial Lecture,[169] where quotations appear from Euler's treatise and other original sources.

In the case of optimum shapes, the first modern study usually cited is Keller's.[170] That author remarks, however, that in the early 1850's Clausen[171] correctly described the tapered column of solid circular section which, for a given length and mass or volume of homogeneous material, sustains the greatest buckling load. As observed by Taylor (Ref. 172, among others), this "strongest column" is also a least-mass design for a prescribed length and load. Taylor's conclusion seems to apply both for columns whose area is permitted to neck down at stations of zero bending moment and for more realistic designs, like those presented here, whose cross section is bounded so as to ensure adequate compressive strength.

Combinations of boundary conditions other than simple support have been studied by Tadjbakhsh and Keller[173] and by Olhoff and Rasmussen.[174] Remarkable singularities are encountered[174] in the case of clamped columns, but the resulting shapes lack practical utility because their locally zero cross sections are ruled out by the infinite compressive stress.

As always, this example calls first for a "preoptimization" step, wherein engineering judgment establishes the topology and other limits on the allowable family of solutions. Prager and Taylor[175] point out that the greatest rigidity is attained within a given area or volume by placing the structural material as close as possible to the outer bounding surface. (Sandwich beams or plates with thin face sheets typify this concept in the case of lateral bending loads.) Since transverse buckling can occur in any direction, the column's ellipse of cross-sectional moments of inertia I should obviously be a circle. This requirement suggested to Keller[170] either a body or revolution or regular polygonal section. The thin-walled hollow circle is a natural choice—although Ref. 170 notes that it is not the absolute best because the I for a given sectional area A may be made unboundedly large if all restrictions on convexity and linear dimensions are removed! For thin-walled circles, squares, etc., it is known that particularly simple minimum-mass solutions can be obtained because both I and the mass per unit length depend linearly on the wall thickness. These will not, however, be elaborated further here.

For ease in the manufacture of truss members, let the designs explored below be limited to solid, convex sections. All these have the property

$$I = kA^2 \tag{A1}$$

with the constant k fixed by the shape. Keller[170] infers the best figure to be an equilateral triangle $[k=(1/6\sqrt{3})=0.096225]$, but the circle $[k=(1/4\pi)=0.079577]$ has other practical advantages. Accordingly, the circle is chosen for illustration—albeit the extension to other values of k is trivial. Under simple support, the load P which buckles a column of length L is related to the mode $w(x)$ of lateral deformation by the homogeneous differential equation

$$EI(x)\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + PL^2 w = 0 \tag{A2}$$

and boundary conditions

$$w(0) = 0 = w(1) \tag{A3}$$

E is Young's modulus, both w and x having been made dimensionless by division with L. It facilitates comparisons if the optimal solution is referred to properties, $(\ldots)_o$, of a circular cylindrical bar with the same buckling load. The familiar formula

$$P = (\pi/L)^2 EI_o \tag{A4}$$

yields its reference area

$$A_o \equiv \pi r_o^2 = 2L\sqrt{P/\pi E} \tag{A5}$$

With $a(x) \equiv A(x)/A_o$, Eq. (A2) can be recast to

$$\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + \frac{\pi^2}{a^2} w = 0 \tag{A6}$$

Let $A_m \equiv A_o a_m$, which depends on P and the material's allowable compressive strength, furnish a lower bound

$$a(x) \geq a_m \tag{A7}$$

Since $a(x)$ is proportional to mass per unit length, the problem calls for choosing this function so as to minimize

$$M \equiv \int_0^1 a(x)\,\mathrm{d}x \tag{A8}$$

subject to constraints (A6), (A7), and conditions (A3).

Among several procedures available, the author prefers the Hamiltonian formulation from variational calculus, which is fully explained in the context of time trajectories by Bryson and Ho (Chaps. 2 and 3, Ref. 3). First Eq. (A6) is cast in state-vector form:

$$\frac{\mathrm{d}w}{\mathrm{d}x} = s \tag{A9}$$

$$\frac{\mathrm{d}s}{\mathrm{d}x} = -\pi^2 \frac{w}{a^2} \tag{A10}$$

Necessary conditions for a local minimum of M are then provided in terms of the Hamiltonian

$$H = a + \lambda_w s - \lambda_s \pi^2 (w/a^2) + \mu[a_m - a] \tag{A11}$$

as follows:

$$\frac{\partial H}{\partial a} = 0 = 1 + 2\pi^2 \frac{\lambda_s w}{a^2} - \mu \tag{A12}$$

$$\frac{\mathrm{d}\lambda_w}{\mathrm{d}x} = -\frac{\partial H}{\partial w} = \pi^2 \frac{\lambda_s}{a^2} \tag{A13}$$

$$\frac{\mathrm{d}\lambda_s}{\mathrm{d}x} = -\frac{\partial H}{\partial s} = -\lambda_w \tag{A14}$$

Minimum-gage constraint (A7) is enforced by the requirement

$$\mu(x) = 1 + \frac{2\pi^2 \lambda_s w}{a^3} \begin{cases} >0, & a = a_m \\ =0, & a > a_m \end{cases} \tag{A15}$$

Another useful relation is derived by eliminating the co-state variable λ_w between Eqs. (A13) and (A14):

$$\frac{\mathrm{d}^2 \lambda_s}{\mathrm{d}x^2} + \frac{\pi^2}{a^2}\lambda_s = 0 \tag{A16}$$

Since there is no preferred direction for x and the maximum bending moments occur near the center, it is reasonable to assume that the fundamental buckling eigenfunction $w(x)$ is symmetrical about $x = ½$ and that $a(x)$ consists of two uniform arcs $a = a_m$, within distances x_m of the ends, matched to a bulged central arc. Under these assumptions, boundary conditions like

$$\frac{\mathrm{d}w(1/2)}{\mathrm{d}x} = 0 = \frac{\mathrm{d}\lambda_s(1/2)}{\mathrm{d}x} \tag{A17}$$

apply. $w(x)$ and $\lambda_s(x)$ satisfy identical differential Eqs. (A6) and (A16), so (A17) shows them to be proportional. Let their relationship be written

$$\lambda_s = -w/2\pi^2 C^3 \tag{A18}$$

where C is a constant to be evaluated. Equations (A12) and (A18) in the unconstrained arc ($\mu = 0$) yield

$$a(x) = (1/C)\,w^{2/3} \tag{A19}$$

Substitution of Eq. (A19) into Eq. (A6) finally produces the uncoupled differential equation

$$\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + \frac{(\pi C)^2}{w^{1/3}} = 0 \tag{A20}$$

Let $w(½) \equiv w_M$ be the (arbitrary) maximum amplitude of the buckling mode. By means of the variable change

$$Z = (w/w_M)^{1/3} \tag{A21}$$

it is straightforward to solve Eq. (A20). Boundary condition (A17) leads to the implicit solution¶¶

$$\left|\frac{1}{2} - x\right| = \frac{\sqrt{3}\,w_M^{2/3}}{2\pi C}\left\{\cos^{-1}\left(\frac{w}{w_M}\right)^{1/3} + \left[\left(\frac{w}{w_M}\right)^{2/3} - \left(\frac{w}{w_M}\right)^{4/3}\right]^{½}\right\} \tag{A22}$$

The tentatively optimal area distribution can be determined from Eq. (A19) after constant C is evaluated. This step is accomplished by matching slope $\mathrm{d}w/\mathrm{d}x$ and deflection w, at points $x = x_m$ and $1 - x_m$, to those in the uniform arcs near the ends. Differential Eq. (A6), with $a = a_m$ and boundary condition (A3), are solved by

$$w = A\sin(\pi x/a_m) \tag{A23}$$

¶¶This result, together with Eq. (A28), may be compared with Eq. (11), Sec. 3, of Keller[170] for the unconstrained column.

in the left-hand arc $0 \le x \le x_m$. The matching process then yields the following three relations among unknowns x_m, C, and the modal amplitude w_m at the matching station:

$$\frac{\sqrt{3}}{\pi} a_m \left(\frac{w_M}{w_m}\right)^{2/3} \left\{ \cos^{-1}\left(\frac{w_m^{1/3}}{w_M^{1/3}}\right) + \sqrt{\left(\frac{w_m}{w_M}\right)^{2/3} - \left(\frac{w_m}{w_M}\right)^{4/3}} \right\} + \frac{2a_m}{\pi}\,\mathrm{ctn}^{-1}\sqrt{3\left[\left(\frac{w_M}{w_m}\right)^{2/3} - 1\right]} = 1 \qquad \text{(A24)}$$

$$x_m = (a_m/\pi)\,\mathrm{ctn}^{-1}\sqrt{3\left[\left(\frac{w_M}{w_m}\right)^{2/3} - 1\right]} \qquad \text{(A25)}$$

$$C = w_m^{2/3}/a_m \qquad \text{(A26)}$$

The sign of μ called for by Eq. (A15) is indeed positive in the uniform arc, because one finds there

$$\mu(x) = 1 - (w/w_m)^2 \ge 0 \qquad \text{(A27)}$$

It is significant that C turns out independent of maximum amplitude w_M and that this quantity also appears only in terms of the ratio w_m/w_M in Eqs. (A24) and (A25). For the remainder of the analysis w_M is therefore taken equal to unity.

A convenient procedure for computational purposes is to start by assuming a series of values for $w_m < 1$. a_m and x_m are then successively determined from Eqs. (A24) and (A25). C from Eq. (A26) into Eq. (A19) then provides the area distribution in the central arc as

$$a(x) = a_m (w/w_m)^{2/3}, \qquad x_m \le x \le 1 - x_m \qquad \text{(A28)}$$

Here $x(w)$ and $w(x)$ must be calculated from Eqs. (A22) and (A26), again with $w_M = 1$. Perhaps the most interesting property of the solution is the performance index M, which compares the presumed minimum mass to that of the uniform cylindrical column with equal buckling load. From Eqs. (A8), (A28), and intermediate formulas for $w(x)$ it can be shown that

$$M = 2a_m x_m + \frac{3\sqrt{3}}{4\pi}\frac{a_m^2}{w_m^{4/3}}\cos^{-1} w_m^{1/3} + \frac{\sqrt{3}}{2\pi} a_m^2 \sqrt{1 - w_m^{2/3}} \left[\frac{3}{2w_m} + \frac{1}{w_m^{1/3}}\right] \qquad \text{(A29)}$$

Figure 28 illustrates the dependence of M on a_m. Values range from $\sqrt{3}/2$ for the unconstrained column ($a_m = 0$)*** to unity at the point $a_m = 1$ where the end load is so great that the requirement on compressive strength begins to dominate the design. Some representative area distributions are plotted on the figure. By way of confirmation of the present results, it is remarked that Haug and Arora[7] employ the simply supported column as an illustrative example in their text, where it is solved both by finite-element methods and by numerical integration of an equation that is equivalent to Eq. (A20) combined with Eqs. (A15) and (A18). For the 11 values of P appearing in Table 6.1 of Ref. 7, data computed by the author agree within an accuracy of better than ±1.5%.

It is clear from Fig. 28 and the preceding development that the dimensionless optimum-column solutions depend only on the single parameter a_m. When these results are to be employed for sizing a physical column of known length L and compressive load P, one must also specify Young's modulus

***Whereas mass savings ranging only up to 13.4% are not dramatic, it is remarked that many other function-space solutions yield better dividends. Compare, for instance, certain optimum torsional cantilevers published by Armand and Vitte.[176]

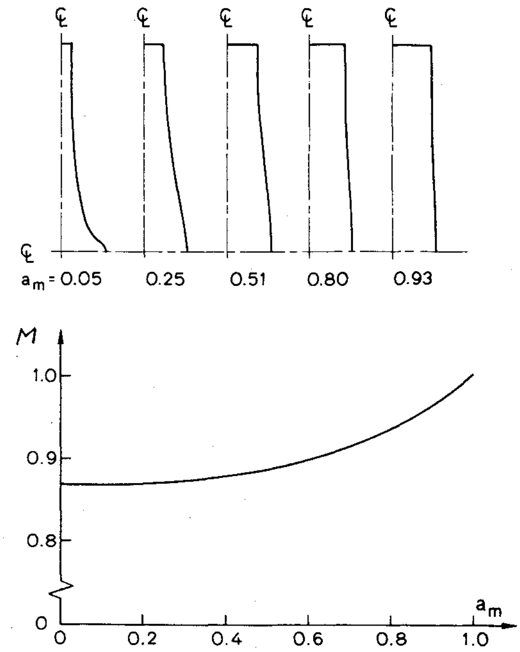

Fig. 28 Dependence of performance index (mass) M on parameter a_m for minimum-mass, solid Euler columns of fixed length and end load. Sketches at top show variations of dimensionless cross-sectional radius with distance from center to one end for five typical columns.

and an allowable stress σ_c for the material. The minimum area is then

$$A_m = P/\sigma_c \qquad \text{(A30)}$$

When divided by A_0 from Eq. (A5), Eq. (A30) yields

$$a_m \equiv A_m/A_0 = \sqrt{\pi E P}/2L\sigma_c \qquad \text{(A31)}$$

Given $a_m < 1$ and A_0, details of the column's dimensions can be readily calculated from Fig. 28 and such formulas as Eqs. (A24), (A25), and (A28).

The questions of *sufficiency* and *uniqueness* of the foregoing optima have not been addressed. First of all, it is remarked that a qualitative examination of solutions obtained from necessary conditions is usually adequate to ensure their practical usefulness and that this appears to be true of the column problem. But exceptions certainly exist, as witness Breakwell and Shoaee.[71] The sufficiency issue is treated, among many others, in Secs. 6.1-6.3 of Bryson and Ho.[3] Based on requiring the second variation of M to be locally positive, the most important requirement is the condition of Legendre and Clebsch, which may be written here as the scalar relation

$$\frac{\partial^2 H}{\partial a^2} > 0, \qquad 0 \le x \le 1 \qquad \text{(A32)}$$

In the central arc, for example, it is easy to derive

$$\frac{\partial^2 H}{\partial a^2} = -\frac{6\pi^2 \lambda_s w}{a^4} = \frac{3}{a} \qquad \text{(A33)}$$

which is clearly positive. In the two uniform arcs, moreover, the second derivative is proportional to w^2 as given by Eq. (A23); the vanishing of this result at the endpoints is not believed to invalidate the sufficiency test.

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References

[1]Wilde, D.J., *Globally Optimal Design,* John Wiley and Sons, N.Y., 1978.

[2]Bliss, G.A., *Calculus of Variations,* The Open Court Publishing Company, LaSalle, Ill., 1925.

[3]Bryson, A.E. Jr. and Ho, Y-C., *Applied Optimal Control,* Hemisphere Publishing Corp., Washington, D.C., 1975 (revised printing).

[4]Leitmann, G., *The Calculus of Variations and Optimal Control,* Plenum Press, N.Y. (to be published; advance copy supplied courtesy of the author).

[5]Fox, R.L., *Optimization Methods for Engineering Design,* Addison-Wesley Publishing Co., Reading, Mass., 1971.

[6]Miele, A., ed., *Theory of Optimum Aerodynamic Shapes,* Vol. 9, Academic Press, N.Y., 1965.

[7]Haug, E.J. and Arora, J.S., *Applied Optimal Design,* John Wiley and Sons, N.Y., 1979.

[8]Beightler, C.S., Phillips, D.T., and Wilde, D.J., *Foundations of Optimization,* 2nd Ed., Prentice-Hall, Inc., N.J., 1979.

[9]Gallagher, R.H. and Zienkiewicz, O.C., eds., *Optimum Structural Design,* John Wiley and Sons, N.Y., 1973.

[10]Pope, G.G. and Schmit, L.A., eds., *Structural Design Applications of Mathematical Programming Techniques,* AGARDograph 149, Technical Editing and Reproduction, Ltd., London, Feb. 1971.

[11]Schmit, L.A. Jr., "Use of Mathematical Programming Methods for Structural Optimization," AGARD Lecture Series No. 70, Los Angeles, Calif., Oct. 1974.

[12]Schmit, L.A. Jr., ed., *Structural Optimization Symposium,* ASME, N.Y., AMD-Vol. 7, 1974.

[13]Niordson, F.I. and Pedersen, P., "A Review of Optimal Structural Design," in *Applied Mechanics,* edited by E. Becker and G.K. Mikhailov, Springer-Verlag, Berlin, 1973, pp. 264-278.

[14]Lev, O., ed., *Structural Optimization - Recent Developments and Applications,* report prepared June 1980 by ASCE Committee on Electronic Computation (to be published by ASCE).

[15]Noor, A.K. and McComb, H.G Jr., ed., "Trends in Computerized Structural Analysis and Synthesis," Special Issue of *International Journal of Computers and Structures,* Vol. 10, No. 1/2, April 1979.

[16]Pierson, B.L., "A Survey of Optimal Structural Design Under Dynamic Constraints," *International Journal for Numerical Methods in Engineering,* Vol. 4, 1972, pp. 491-499.

[17]Stroud, W.J., "Automated Structural Design with Aeroelastic Constraints: A Review and Assessment of the State of the Art," *Structural Optimization,* AMD, Vol. 7, ASME, Nov. 1974, pp. 77-118.

[18]Koopmans, T.C., *Three Essays on the State of Economic Science,* McGraw-Hill, N.Y., 1957 (see especially "Allocation of Resources and the Price System").

[19]Schumann, J., *Input-Output-Analyse,* Springer-Verlag, Berlin, 1968.

[20]Rudd, D.F. and Watson, C.C., *Strategy of Process Engineering,* John Wiley and Sons, N.Y., 1968.

[21]Dantzig, G.B., *Linear Programming and Extensions,* Princeton Univ. Press, Princeton, N.J., 1963.

[22]Ragsdell, K.M., "The Evaluation of Optimization Software for Engineering Design," The Mathematical Programming Society Meeting: Testing and Validating Algorithms and Software, Boulder, Colo., Jan. 1981.

[23]Howell, L.J., "Automotive and Aerospace Design—A Convergence of SDM Interests," *AIAA Journal,* Vol. 18, Nov. 1980, pp. 40-59.

[24]Bennett, J.A. and Nelson, M.F., "An Optimization Capability for Automotive Structures," *SAE Transactions,* Vol. 88, 1979, pp. 3236-3244.

[25]Bennett, J.A., Lin, K-H., and Nelson, M.F., "The Application of Optimization Techniques to Problems of Automotive Crashworthiness,"*SAE Transactions,* Vol. 86, 1977, pp. 2255-2262.

[26]Ragsdell, K.M., "Optimization as a Tool for Automotive Design," SAE Technical Paper 800432, Feb. 1980.

[27]McFarland, M.W., ed., *The Papers of Wilbur and Orville Wright,* Vols. I and II, Arno Press Reprint Edition, N.Y., 1972.

[28]Baker, M.P. "Wright Brothers as Aeronautical Engineers," *SAE Quarterly Transactions,* Vol. 5, Jan. 1951, pp. 1-11 (see also discussions, pp. 11-17).

[29]Combs, H. B. (with M. Caidin), *Kill Devil Hill,* Houghton Mifflin Co., Boston, 1979.

[30]Koppen, O.C., private communication.

[31]Schmit, L.A. Jr., "Structural Design by Systematic Synthesis," *Proceedings ASCE 2nd Conference on Electronic Computation,* Pittsburgh, Penn., Sept. 1960, pp. 105-132.

[32]Schmit, L.A. Jr., "Structural Synthesis from Abstract Concept to Practical Tool," *Proceedings of the AIAA/ASME 18th SDM Conference,* Los Angeles, March 1977.

[33]Stadler, W., "A Survey of Multicriteria Optimization or the Vector Maximum Problem, Part I: 1776-1960," *Journal of Optimization Theory and Applications,* Vol. 29, No. 1, Sept. 1979, pp. 1-52.

[34]Petiau, C. and Lecina, G., "Elements Finis et Optimisation des Structures Aéronautiques," *AGARD Conference Proceedings No. 280—The Use of Computers as a Design Tool,* pp. 23-1 - 23-16.

[35]Pedersen, P., "On the Minimum Mass Layout of Trusses," *Symposium on Structural Optimization,* edited by R.A. Gellatly and G.G. Pope, AGARD Conference Proceedings No. 36, Oct. 1969.

[36]Housner, G.W. and Hudson, D.E., *Applied Mechanics: Statics,* 2nd Ed., D. van Nostrand Co., Princeton, N.J., 1961.

[37]Michell, A.G.M., "The Limits of Economy of Material in Frame Structures," *Philosophical Magazine,* Ser. 6, Vol. 8, No. 47, Nov. 1904, pp. 589-597.

[38]Hemp, W.S., "Studies in the Theory of Michell Structures," *Proceedings of the 11th International Congress of Applied Mechanics,* Munich, 1964, Springer-Verlag, Berlin, 1966, pp. 621-628.

[39]Sobieski, J. and Bhat, R.B., "Adaptable Structural Synthesis Using Advanced Analysis and Optimization Coupled by a Computer Operating System," *Journal of Aircraft,* Vol. 18, Feb. 1981, pp. 142-149.

[40]Hajela, P. and Sobieszczanski-Sobieski, J., "The Controlled Growth Method—A Tool for Structural Optimization," *Proceedings of the AIAA/ASME/ASCE/AHS 22nd Structures, Structural Dynamics and Materials Conference,* Atlanta, Ga., April 1981, pp. 206-215.

[41]Eggers, A.J. Jr., Resnikoff, M.M., and Dennis, D.H., "Bodies of Revolution Having Minimum Drag at High Supersonic Airspeeds," NACA Rept. 1306, 1957.

[42]Prandtl, L., "Applications of Modern Hydrodynamics to Aeronautics," translated as NACA Rept. 116 (original sources dated 1918-1920).

[43]*Flight,* Vol. XXX, No. 1455, Nov. 12, 1936, p. 513.

[44]Goldstein, S., "On the Vortex Theory of Screw Propellers," *Proceedings of the Royal Society, Series A,* London, Vol. 123, No. 440, 1929.

[45]Theodorsen, T., *Theory of Propellers,* McGraw-Hill, N.Y., 1948.

[46]Craig, R.E., private communication, Convair Div., General Dynamics Corp.

[47]" 'Wasp-Waisted' Transonic Fighters," *Interavia,* Vol. X, No. 12, Dec. 1955, pp. 928-930.

[48]Whitcomb, R.T., "A Study of the Zero-Lift Drag-Rise Characteristics of Wing-Body Combinations Near the Speed of Sound," NACA Rept. 1273, 1955.

[49]Lomax, H. and Heaslet, M.A., "Recent Developments in the Theory of Wing-Body Wave Drag," *Journal of the Aeronautical Sciences,* Vol. 23, Dec. 1956, pp. 1061-1074.

[50]Hague, D.S., Rozendaal, H.L., and Woodward, F.A., "Application of Multivariable Search Techniques to Optimal Aerodynamic Shaping Problems," *Journal of the Astronautical Sciences,* Vol. 15, No. 6, Nov./Dec. 1968, pp. 283-296.

[51]Harvey, H.P. and Johnson, R.R., "Application of Numerical Optimization to the Design of Wings with Specified Pressure Distributions," NASA CR 3238, Feb. 1980.

[52]Hicks, R.M. and Henne, P.A., "Wing Design by Numerical Optimization," *Journal of Aircraft,* Vol. 15, July 1978, pp. 407-412.

[53]Jameson, A., Caughey, D.A., Newman, P.A., and Davis, R.M., "A Brief Description of the Jameson-Caughey NYU Transonic Swept-Wing Computer Program—FLO22," NASA TM X-73,996, Dec. 1976.

[54]Wilkinson, K.W., private communication, Grumman Aerospace Corp.

[55]Mason, W., "Computation of the Minimum Trimmed Drag Attainable on Complex Configurations and the Associated Design Spanload," Memo. No. EG-ARDYN-78-44, Grumman Aerospace Corp., May 1978.

[56]Lamar, J.E., "A Vortex Lattice Method for the Mean Camber Shapes of Trimmed Non-Coplanar Planforms with Minimum Vortex Drag," NASA TND-8090, June 1976.

[57]Garberoglio, J.E., Song, J.O., and Boudreaux, W.L., "Optimization of Compressor Vane and Bleed Settings," (to be published as an AFAPL-TR, 1981).

[58]Vanderplaats, G.N., "CONMIN—A FORTRAN Program for Constrained Function Minimization; User's Manual," NASA TM X-62, 282, Aug. 1973.

[59]Rao, S.S. and Gupta, R.S., "Optimum Design of Axial Flow Gas Turbine Stage, Part I: Formulation and Analysis of Optimization Problem," *ASME Journal of Engineering for Power,* Vol. 102, Oct. 1980, pp. 782-789.

[60]Rao, S.S. and Gupta, R.S., "Optimum Design of Axial Flow Gas Turbine Stage, Part II: Solution of the Optimization Problem and Numerical Results," *ASME Journal of Engineering for Power,* Vol. 102, pp. 790-797.

[61]Hearsey, R., "Study of 11-ft Transonic Wind Tunnel Energy Consumption," Rept. No. HT7907, Hearsey Technology, June 1979.

[62]Ospring, M.J., private communication, NASA Ames Research Center.

[63]Hague, D.S., Reichel, R.H., Jones, R.T., and Glatt, C.R., "Optimizing a Liquid Propellant Rocket Engine with an Automated Combustor Design Code—AUTOCOM," NASA CR-120856, 1971.

[64]DeHoff, R.L., Hall, W.E. Jr., Adams, R.J., and Gupta, N.K., "F-100 Multivariable Control Synthesis Program," AFAPL-TR-77-35, Vols. I and II, June 1977.

[65]Hill, R.J. and Henderson, R.E., "Durable Propulsion Means More Air Power," *Astronautics & Aeronautics,* Vol. 18, June 1980, pp. 54-61.

[66]McLean, G.D., "Some of the Benefits and Difficulties of Automated Flight Planning," *Proceedings of the Technical Conference on Automated Flight-Planning, Communications and Control,* Airways Engineering Society, April 1965, published by Bradcon, Inc., Newark, N.J.

[67]McLean, G.D. and Cosgrove, J.C., "IBM 1620 Data Processing System Flight Planning Program," 21st Annual National Conference, SAWE, Technical Paper 351, Seattle, Wash., May 1962.

[68]Simpson, L., Bashioum, D.L., and Carr, E.C., "Computer Flight Planning in the North Atlantic," *Journal of Aircraft,* Vol. 2, July/Aug. 1965, pp. 337-346.

[69]Ashley, H., *Engineering Analysis of Flight Vehicles,* Addison-Wesley Publishing Co., Reading, Mass., 1974.

[70]Speyer, J.L., "Non-Optimality of the Steady-State Cruise for Aircraft," *AIAA Journal,* Vol. 14, Nov. 1976, pp. 1604-1610.

[71]Breakwell, J.V. and Shoaee, H., "Minimum Fuel Flight Paths for Given Range," AIAA/AAS Astrodynamics Conference, Danvers, Mass., Aug. 1980.

[72]Erzberger, H. and Lee, H.Q., "Constrained Optimum Trajectories with Specified Range," *Journal of Guidance and Control,* Vol. 3, No. 1, Jan.-Feb. 1980, pp. 78-85.

[73]Lee, H.Q. and Erzberger, H., "Algorithm for Fixed-Range Optimal Trajectories," NASA Technical Paper 1565, July 1980.

[74]Schultz, R.L. and Zagalsky, N.R., "Aircraft Performance Optimization," *Journal of Aircraft,* Vol. 9, Feb. 1972, pp. 108-114.

[75]Calise, A.J., "Extended Energy Management Methods for Flight Performance Optimization," *AIAA Journal,* Vol. 15, March 1977, pp. 314-321.

[76]Burrows, J.W., "Economy Cruise Strategies," Document No. D6-48902TN, Boeing Commercial Airplane Co., Oct. 1980.

[77]Falco, M. and Kelley, H.J., "Aircraft Symmetric Flight Optimization," *Control and Dynamic Systems,* Vol. 10, 1973, pp. 89-129.

[78]Bryson, A.E. Jr. and Denham, W.F., "A Steepest-Ascent Method for Solving Optimum Programming Problems," *ASME Journal of Applied Mechanics,* Vol. 29, No. 2, June 1962, pp. 247-257.

[79]Landgraf, S.K., "Some Applications of Performance Optimization Techniques to Aircraft," *Journal of Aircraft,* Vol. 2, March/April 1965, pp. 153-154.

[80]Hague, D.S., "Three Degree-of-Freedom Problem Optimization Formulation," USAF FDL-TDR-64-1, Part 1, Vol. 3, 1964.

[81]Hague, D.S., private communication, Aerophysics Research Corp.

[82]Rutowsky, E.S., "Energy Approach to the General Aircraft Performance Problem," *Journal of the Aerospace Sciences,* Vol. 21, March 1954, pp. 187-195.

[83]Bryson, A.E. Jr., Desai, M.N., and Hoffman, W.C., "Energy-State Approximation in Performance Optimization for Supersonic Aircraft," *Journal of Aircraft,* Vol. 6, Nov./Dec. 1969, pp. 481-488.

[84]Leitmann, G., "The Optimization of Rocket Trajectories – A Survey," *Progress in the Astronautical Sciences,* edited by S. F. Singer, North-Holland Publishing Co., Amsterdam, 1962, Vol. I, Chap. IV, pp. 147-213.

[85]Malina, F.J. and Summerfield, M., "The Problem of Escape from the Earth by Rocket," *Journal of the Aeronautical Sciences,* Vol. 14, 1947, pp. 471-480.

[86]Brusch, R.G., "Trajectory Optimization for the Atlas/Centaur Launch Vehicle," *Journal of Spacecraft and Rockets,* Vol. 14, Sept. 1977, pp. 550-555.

[87]Tramonti, L., "Launch Vehicle Trajectory Design for the FLTSATCOM Mission," Rept. No. CASD/LVP77-063, Convair Division, General Dynamics, Sept. 1977.

[88]Lanflisi, R. private communication, Convair Division, General Dynamics Corp.

[89]Rheinfurth, M., private communication, NASA Marshall Spaceflight Center.

[90]Brusch, R.G., "A Nonlinear Programming Approach to Space Shuttle Trajectory Optimization," *Journal of Optimization Theory and Applications,* Vol. 13, Jan. 1974, pp. 94-118.

[91]Johnson, I.L., "Optimization of the Solid-Rocket Assisted Space Shuttle Ascent Trajectory," *Journal of Spacecraft and Rockets,* Vol. 12, Dec. 1975, pp. 765-769.

[92]Isaacs, R., *Differential Games,* John Wiley and Sons, N.Y., 1965.

[93]Merz, A.W. and Hague, D.S., "Coplanar Tail-Chase Aerial Combat as a Differential Game," *AIAA Journal,* Vol. 15, Oct. 1977.

[94]Breakwell, J.V. and Merz, A.W., "Minimum Required Capture Radius in a Coplanar Model of the Aerial Combat Problem," *AIAA Journal,* Vol. 15, Aug. 1977, pp. 1089-1094.

[95]Tonkinson, J.A., private communication, Northrop Corp.

[96]Hague, D.S. et al., "Combat Optimization and Analysis Program," in USAF AFFDL-TR-72-57, Vol. II, 1972 (title Unclassified, report Classified); see also AFWAL-TR-80-3089, 1980, Classified.

[97]Boyd, J.R. and Christie, T.P., "Expanded Energy Maneuverability Concept," Rept. SAGF 69-159, Oct. 1969 (declassified Dec. 31, 1975; available as Defense Documentation Ctr. No. F33657-70-C-0717).

[98]Patierno, J.H., Stevens, J.R., and Evanbar, M.S., "Application of Optimum Three-Dimensional Maneuvering Techniques to Parametric Aircraft Design Tradeoffs," in USAF AFFDL-TR-72-57, Vol. II, pp. 57-81 (Unclassified).

[99]Anderson, D.C., Berger, R.L., and Hess, J.R. Jr., "Maneuver Load Control and Relaxed Static Stability Applied to a Contemporary Fighter Aircraft," *Journal of Aircraft,* Vol. 10, Feb. 1973, pp. 112-120.

[100]Ormand, J.F., Hines, J.D., and Hide, J.P., "Improvements to a Computer Program for Optimizing a 3-D Engagement Between a Surface to Air Missile and Tactical Aircraft," a Frank J. Seiler Report No. SRL-TR-72-0003, U.S.A.F. Academy, Colo., Feb. 1972, A.D. 891929.

[101]Vachino, R.F., Schaefer, J.F., deDoes, D.H., Morgan, B.S. Jr., and Cook, G., "The Application of the Method of Steepest Descent to a Pursuit-Evasion Problem," *Proceedings of the Guidance, Control and Flight Dynamics Conference,* Pasadena, Calif., Aug. 1968.

[102]Shinar, J. and Gutman, S., "Three-Dimensional Optimal Pursuit and Evasion with Bounded Controls," *IEEE Transactions on Automatic Control,* Vol. AC-25, June 1980, pp. 492-496.

[103]Calise, A.J., "Optimal Thrust Control with Proportional Navigation Guidance," *Journal of Guidance and Control,* Vol. 3, No. 4, July-Aug. 1980, pp. 312-318.

[104]Draper, C.S. and Li, Y.T., "Principles of Optimalizing Control Systems and an Application to the Internal Combustion Engine," publication sponsored by M.I.T. Aeronautical Engineering Dept., published by ASME, N.Y., Sept. 1951.

[105]Berman, H. "Improved Flight Control Performance and Failure Tolerance Using Modern Control Techniques," *Proceedings of the 12th Congress of the International Council of Aeronautical Sciences,* Munich, Oct. 1980, pp. 209-218, .

[106]Blight, J.D. and Gangsaas, D., "Classical and Modern Control Design of a Speed Hold System for a STOL Airplane," AIAA Paper 81-0017, Jan. 1981.

[107]Bryson, A.E. Jr. and Hall, W.E. Jr., "Optimal Control and Filter Synthesis by Eigenvector Decomposition," Stanford University, Dept. of Aeronautics and Astronautics, Internal Report SUDAAR 436, Dec. 1971.

[108]Tabak, D., Schy, A.A., Giesy, D.P., and Johnson, K.G., "Application of Multiobjective Optimization in Aircraft Control Systems Design," *Automatica,* Vol. 15, 1979, pp. 595-600.

[109]Michael, G.J. and Farrar, F.A., "The Use of Model Following Methods to Generate Suboptimal Feedback Control," *International Journal of Control,* Vol. 17, 1973, pp. 1325-1333.

[110]Widnall, W.S., "The Minimum-Time Thrust-Vector Control Law in the Apollo Lunar-Module Autopilot," *Automatica,* Vol. 6, 1970, pp. 661-672.

[111]Kalman, R.E., "A New Approach to Linear Filtering and Prediction Problems," *Transactions of ASME, Series D, Journal of Basic Engineering,* Vol. 82, No. 1, March 1960, pp. 35-45.

[112]Schmidt, S.F., "The Kalman Filter: Its Recognition and Development for Aerospace Applications," *Journal of Guidance and Control,* Vol. 4, No. 1, Jan.-Feb. 1981, pp. 4-7.

[113]Schmidt, S.F., Weinberg, J.D., and Lukesh, J.S., "Case Study of Kalman Filtering in the C-5 Aircraft Navigation Systems," IEEE Case Studies Seminar, Ann Arbor, Mich., June 1968; see also AGARDograph 139.

[114]Bryson, A.E. Jr., private communication, Stanford University.

[115]Newsom, J.R., Abel, I., and Dunn, H.J., "Application of Two Design Methods for Active Flutter Suppression and Wind-Tunnel Test Results," NASA TP-1653, May 1980.

[116]Murrow, H.N. and Eckstrom, C.V., "Drones for Aerodynamic and Structural Testing (DAST)-A Status Report," *Journal of Aircraft,* Vol. 16, Aug. 1979, pp. 521-526.

[117]Newsom, J.R., "A Method for Obtaining Practical Flutter-Suppression Control Laws Using Results of Optimal Control Theory," NASA TP-1471, Aug. 1979.

[118]Fletcher, R. and Powell, M.J.D., "A Rapidly Convergent Descent Method for Minimization," *Computer Journal,* Vol. 6, No. 2, July 1963, pp. 163-168.

[119]Nissim, E., "Flutter Suppression Using Active Controls Based on the Concept of Aerodynamic Energy," NASA TN-D-6199, 1971.

[120]Schmit, L.A. Jr. and Miura, H., "Approximation Concepts for Efficient Structural Synthesis," NASA CR-2552, March 1976.

[121]Fleury, C. and Geradin, M., "Optimality Criteria and Mathematical Programming in Structural Weight Optimization," *Journal of Computers and Structures,* Vol. 8, Jan. 1978, pp. 7-17.

[122]Rizzi, P., "Optimization of Multi-Constrained Structures Based on Optimality Criteria," *Proceedings of the AIAA/ASME/SAE 17th Structures, Structural Dynamics and Materials Conference,* King of Prussia, Penn., May 1976, pp. 448-462.

[123]Venkayya, V.B. "Design of Optimum Structures," *Journal of Computers and Structures,* Vol. 1, No. 1-2, Jan. 1971, pp. 265-309.

[124]"C-5A Proposal-Airframe Subsystem," TSAF No. U-5-108-14, 0(ASD), Vol. IV, "Structural Design and Description," Document No. 2-19-4-2, 1965, pp. VI-23 to VI-37.

[125]Biggers, S.B., private communication, Lockheed Georgia Co.

[126]Melcon, M.A. and Ensrud, A.F., "Analysis of Stiffened Curved Panels Under Shear and Compression," *Journal of the Aeronautical Sciences,* Vol. 20, Feb. 1953, pp. 111-119, 126.

[127]Schmit, L.A. and Mehrinfar, M., "Multilevel Optimum Design of Structures with Fiber Composite Stiffened-Panel Components," *Proceedings of the AIAA/ASME/ASCE/AHS 21st Structures, Structural Dynamics and Materials Conference,* Seattle, Wash., May 1980.

[128]Haftka, R.T. and Starnes, J.H., "Application of a Quadratic Extended Penalty Function for Structural Optimization," *AIAA Journal,* Vol. 14, June 1976, pp. 718-724.

[129]Haftka, R.T., "Automated Procedure for Design of Wing Structures to Satisfy Strength and Flutter Requirements," NASA TN D-7264, 1973.

[130]Sobieski, J.S., Barthelemy, J.F., and Riley, K.M., "Sensitivity of Optimum Solutions to Problem Parameters," *Proceedings of the AIAA/ASME/ASCE/AHS 22nd Structures, Structural Dynamics and Materials Conference,* Atlanta, Ga., April 1981.

[131]Ashley, H. and McIntosh, S.C. Jr., "Application of Aeroelastic Constraints in Structural Optimization," *Proceedings of the 12th International Congress of Applied Mechanics,* Aug. 1968, Springer-Verlag, Berlin, pp. 100-113.

[132]Weisshaar, T.A., "Aeroelastic Optimization of a Panel in High Mach Number Supersonic Flow," *Journal of Aircraft,* Vol. 9, Sept. 1972, pp. 611-617.

[133]Johnson, E.H., Rizzi, P., Ashley, H., and Segenreich, S.A., "Optimization of Continuous One-Dimensional Structures Under Steady Harmonic Excitation," *AIAA Journal,* Vol. 14, Dec. 1976, pp. 1690-1698.

[134]Segenreich, S.A., Johnson, E.H., and Rizzi, P., "Three Contributions to Minimum Weight Structural Optimization with Dynamic and Aeroelastic Constraints," SUDAAR No. 501, Stanford Univ., Dept. of Aeronautics and Astronautics, Aug. 1976.

[135]Turner, M.J., "Proportioning Members of a Structure for Maximum Stiffness with Given Weight," Report ANC-5, Vought-Sikorsky Aircraft Co., Jan. 1942.

[136]Siegel, S., "A Flutter Optimization Program for Aircraft Structural Design," AIAA Paper 72-795, Aug. 1972.

[137]Hassig, H.J., private communication, Lockheed California Co.

[138]O'Connell, R.F., Radovcich, N.A., and Hassig, H.J., "Structural Optimization with Flutter Speed Constraints Using Maximized Step Size," *Journal of Aircraft,* Vol. 14, Jan. 1977, pp. 85-89.

[139]O'Connell, R.F., Hassig, H.J., and Radovcich, N.A., "Study of Flutter Related Computational Procedures for Minimum Weight Structural Sizing of Advanced Aircraft," NASA CR-2607, March 1976.

[140]O'Connell, R.F., Hassig, H.J., and Radovcich, N.A., "Study of Flutter Related Computational Procedures for Minimum Weight Structural Sizing of Advanced Aircraft—Supplemental Data," NASA CR-132722, Aug. 1975.

[141]Greene, W.H. and Sobieski, J.S., "Minimum Mass Sizing of a Large Low-Aspect Ratio Airframe for Flutter-Free Performance," NASA TM-81818, May 1980.

[142]Wilkinson, K., Lerner, E., and Taylor, R.F., "Practical Design of Minimum-Weight Aircraft Structures for Strength and Flutter Requirements," *Journal of Aircraft,* Vol. 13, Aug. 1976, pp. 614-624.

[143]"Application of FASTOP to the Flutter and Strength Design of Military Flight Vehicles," AFFDL-TR-77-28, Vol. I—F-16 Wing/Store Flutter, May 1977, and Vol. II—Design of an Advanced Fighter Wing, June 1977.

[144]Isakson, G., Pardo, H., Lerner, E. and Venkayya, V.B., "ASOP-3, A Program for Optimum Structural Design to Satisfy Strength and Deflection Constraints," *Journal of Aircraft,* Vol. 15, July 1978, pp. 422-428.

[145]McCullers, L.A. and Lynch, R.W., "Composite Wing Design for Aeroelastic Requirements," *Proceedings of Conference on Fibrous Composites in Flight Vehicle Design,* AFFDL-TR-72-130, Sept. 1972, pp. 951-972.

[146]Lynch, R.W., Rogers, W.A. and Braymen, W.W., "Aeroelastic Tailoring of Advanced Composite Structures for Military Aircraft, Volume I—General Study," General Dynamics Corporation, Ft. Worth, Tex., Final Rept., F33615-75-C-3061, April 1977.

[147] Lynch, R.W., Rogers, W.A., and Braymen, W.W., "Aeroelastic Tailoring of Advanced Composite Structures for Military Aircraft, Volume II—Wing Preliminary Design," General Dynamics Corp., Ft. Worth, Tex., Final Rept., F33615-75-C-3061, April 1977.

[148] Lynch, R.W., private communication, Ft. Worth Div., General Dynamics Corp.

[149] Price, M.A., "HiMAT Structural Development Design Methodology," NASA Rept., 144886, Oct. 1979.

[150] Brown, L.E. Jr., Price, M.A., and Gingrich, P.B., "Aeroelastically Tailored Wing Design," *Proceedings of Evolution of Aircraft Wing Design Symposium,* Dayton, Ohio, March 1980, pp. 141-146.

[151] Ellis, J.W., "Development and Hardware Verification of an Aeroelastically Tailored Forward Swept Wing," *Proceedings of the 26th National SAMPE Symposium,* Los Angeles, April 1981.

[152] Ascani, L., Burroughs, B., Lackman, L., and O'Brien, W., "Advanced Structures; Meeting the Challenge of Low Cost Future Aircraft Systems," Aug. 1980.

[153] Triplett, W.E., "Aeroelastic Tailoring of a Forward Swept Wing and Comparisons with Three Equivalent Aft Swept Wings," *Proceedings of the 21st AIAA/ASME SDM Conference,* Seattle, Wash., May 1980, pp. 754-760.

[154] Hague, D.S. and Glatt, C.R., "Application of Multivariable Search Techniques to the Optimal Design of a Hypersonic Cruise Vehicle," NASA CR-73202, April 1968.

[155] Thelander, J.A., "Use of Variational Analysis in Aerospacecraft Design," *Journal of Aircraft,* Vol. 4, May-June 1967, pp. 178-185.

[156] Silver, B.W. and Ashley, H., "Optimization Techniques in Aircraft Configuration Design," SUDAAR No. 406, Stanford Univ., Dept. of Aeronautics and Astronautics, June 1970.

[157] Ardema, M.D., Harper, M., Smith, C.L., Waters, M.H., and Williams, L.J., "Conceptual Design of Reduced Energy Transports," *Journal of Aircraft,* Vol. 13, Aug. 1976, pp. 545-550.

[158] Sliwa, S.M., "Use of Constrained Optimization in the Conceptual Design of a Medium-Range Subsonic Transport," NASA TP-1762, Dec. 1980.

[159] Sobieski, J.S., McCullers, L.A., Ricketts, R.H., Santoro, N.J., Beskenis, S.D., and Durtze, W.L., "Structural Design Studies of a Supersonic Cruise Arrow Wing Configuration," NASA CP-001, Nov. 1976.

[160] Silver, B.W., "Optimization Studies in Aircraft Design," SUDAAR No. 426, Stanford Univ., Dept. of Aeronautics and Astronautics, June 1971.

[161] Jensen, S.C., Rettie, I.H., and Barber, E.A., "Role of Figures of Merit in Design Optimization and Technology Assessment," *Journal of Aircraft,* Vol. 18, Feb. 1981, pp. 76-80.

[162] Jensen, S.C., private communication, Boeing Military Airplane Co.

[163] Pontryagin, L.S. et al., *The Mathematical Theory of Optimal Processes,* translated from the Russian, Interscience Publishers, N.Y., 1962.

[164] Morris, A.J., Bartholomew, P., and Dennis, J., "A Computer Based System for Structural Design, Analysis and Optimization," *AGARD Conference Proceedings, No. 280: The Use of Computers as a Design Tool,* 1979, pp. 20-1-20-25.

[165] Schneider, G., Gödel, H., and Sensburg, O., "Structural Optimization of Advanced Aircraft Structures," *Proceedings of the ICAS Congress,* Munich, Oct. 1980.

[166] Alsaigh, J.M. et al., "Optimization of Steel Transmission Poles," *Journal of the University of Kuwait* (Science), Vol. 7, 1980, pp. 95-108 (in English with Arabic summary).

[167] Chin Fan, "How to be Efficient: A Simple Introduction to 'Optimization,' " *Hang K'ung Chih Shik,* No. 4, pp. 33-35, about 1978, translated as FTD-ID(RS) T-2108-78 by USAF Foreign Technology Division.

[168] Vanderplaats, G.N., "Structural Optimization—Past, Present and Future," AIAA Paper 81-0897, May 1981.

[169] Hoff, N.J., "Buckling and Stability," 41st Wilbur Wright Memorial Lecture, *Journal of the Royal Aeronautical Society,* Vol. 58, Jan. 1954, pp. 3-52.

[170] Keller, J.B., "The Shape of the Strongest Column," *Archive for Rational Mechanics and Analysis,* Vol. 5, 1960, pp. 275-285.

[171] Clausen, T., "Ueber die Form Architektonischer Saulen," *Bulletin Physico-Mathematiques et Astronomiques,* Tome 1, 1849-1853, St. Petersburg, 1953, pp. 279-294.

[172] Taylor, J.E., "Optimum Design of a Vibrating Bar with Specified Minimum Cross Section," *AIAA Journal,* Vol. 6, July 1968, pp. 1379-1381.

[173] Tadjbakhsh, I. and Keller, J.B., "Strongest Columns and Isoperimetric Inequalities for Eigenvalues," *ASME Journal of Applied Mechanics,* Vol. 29, No. 1, March 1962, pp. 159-164.

[174] Olhoff, N. and Rasmussen, S.H., "On Single and Bimodal Optimum Buckling Loads of Clamped Columns," Rept. No. 111, Danish Center for Applied Mathematics and Mechanics, Oct. 1976.

[175] Prager, W. and Taylor, J.E., "Problems of Optimal Structural Design," *ASME Journal of Applied Mechanics,* Vol. 35, No. 1, March 1968, pp. 102-106.

[176] Armand, J.L. and Vitte, W.J., "Foundations of Aeroelastic Optimization and Some Applications to Continuous Systems," Stanford University, Dept. of Aeronautics and Astronautics Internal Report, SUDAAR No. 390, Jan. 1970.

[177] von Kármán, T. and Edson, L., *The Wind and Beyond,* Little Brown and Co., Boston, 1967, p. 347.